

Ideology Algorithm: A Socio-inspired Optimization Method

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Abstract

This paper introduces a new socio-inspired metaheuristic technique referred to as Ideology Algorithm (IA). It is inspired by the self-interested and competitive behaviour of political party individuals which makes them improve their ranking. IA demonstrated superior performance as compared to other well-known techniques in solving unconstrained test problems. Wilcoxon Signed-Rank Test, as a statistical tool, is applied to verify the performance of IA in solving optimization problems. The results are compared with seven well known and some recently proposed optimization algorithms (PSO, CLPSO, CMAES, ABC, JDE, SADE and BSA). A total of 75 unconstrained benchmark problems are used to test the performance of IA up to 30 dimensions. The results from this study show that IA outperforms the other algorithms in terms of number of fitness function evaluations and run time. The eminent observed features of the algorithm are also discussed.

Keywords: metaheuristic, ideology algorithm, socio-inspired optimization, unconstrained test problems

1. Introduction and Motivation

Over the last two decades, meta-heuristic optimization techniques have become increasingly popular and essential in applied mathematics [1-3]. Optimization algorithms are functioning as to find the best values for system variables under various conditions. Some well-known metaheuristics such as Particle Swarm Optimization (PSO) [4], Genetic Algorithm (GA) [5], Ant Colony Optimization (ACO) [6] are fairly well-known and they are applied in various fields of study. With regard to some drawbacks of classical optimization strategies as well as to achieve simplicity, flexibility and derivation-free mechanism, several metaheuristics have been designed [7–12].

Metaheuristics are inspired by simple concepts. They are usually related to physical phenomenon, animal's behaviour and evolutionary concepts. The simplicity allows people to simulate different natural concepts, propose new meta-heuristics, propose hybridization and improved versions. Second, the applicability of metaheuristics to a variety of problems without significant changes in the algorithms' structure makes them flexible. In other words, little problem specific information is required. Also, different techniques could be deployed to support the algorithm solving a variety of class of problems. Third, the majority of metaheuristics have derivation-free mechanisms. Metaheuristics finds the solutions stochastically in contrast with gradient-based optimization techniques. The process of optimization starts with random solution(s), and the calculation of derivative for search spaces are not needed. This makes metaheuristics appropriate to apply in real world problems. Finally, the ability of metaheuristics to avoid local optima makes them reach quickly in the close neighbourhood of the region where the global optimum could be potentially located.

Generally, metaheuristics can be classified into three main classes: evolutionary, physics-based, and swarm intelligence (SI) algorithms. Evolutionary algorithms (EA) are inspired by the concepts of evolution in nature. When the objective function for an optimization problem is non-linear and non-differentiable, EA techniques are typically used to find the global optimum [13–15]. EAs have been applied for various real world engineering problems such as reverse engineer causal networks [16], commercial computer-automated exterior lighting design [17], Nano-scale crossbar architectures [18], dynamic stochastic districting and routing problem [19], neural network classifier [20], assembly line configurations [21], configurations of mobile applications [22], electric power distribution networks [23], word sense disambiguation problem [24], surgery scheduling problems [25], image processing [26], speech recognition [27] and many other problems.

The most commonly used EA optimization techniques are based on swarm intelligence (SI) [15, 28]. SI is characterized by its unique mechanism which mimics the behaviour of swarms of social insects, flocks of birds and schools of fish. The benefits of these approaches as compared with conventional techniques are the flexibility and robustness. These properties make SI a successful design paradigm for algorithms to deal with increasingly complex problems [29]. The PSO simulates the social behaviour as a representation of the movement of organisms in the school of fish [30]. The comprehensive learning particle swarm optimizer (CLPSO) [32] and PSO2011 [33] are advanced versions of the standard PSO [4]. The ACO algorithm is proposed based on strategies of ants in accessing food sources [6]. In Artificial

Bee Colony (ABC) algorithm, the nature behaviour of honey bees in discovering food sources are mimicked [15]. The Cuckoo Search (CS) algorithm is based on the behaviour of cuckoo species by laying their eggs in other host birds' nests [31]. A recently proposed algorithm, named Covariance Matrix Adaptation Evolution Strategy (CMAES) [36], is based on basic genetic rules. The Differential Evolution (DE) algorithm [34, 35] is a population-based stochastic function minimizer. The adaptive differential evolution algorithm (JDE) [37], the parameter adaptive differential evolution algorithm (JADE) [38] and the self-adaptive differential evolution algorithm (SADE) [35] are DE's advanced versions. Another recently proposed algorithm, Backtracking Search Algorithm (BSA), is acting as a function optimizer by generating a trial individual using basic genetic operators (selection, mutation and crossover). A non-uniform crossover strategy, which is more complex than the crossover strategies used in many genetic algorithms, is used by BSA [39].

The work in this paper is motivated from ideologies which are existed in human society for ages. In the context of politics, there are numerous kind of ideologies such as conservative, socialism, left-wing, right-wing, democratic, republic, communism, etc. There are several political parties exist in the world which follow these ideologies in different forms. For example, Conservative Party and Labour Party (United Kingdom), Republican Party and Democratic Party (United States of America), Communist Party (China), and Bharatiya Janata Party (India).

This paper introduces a novel socio-inspired algorithm referred to as Ideology Algorithm (IA). The society individuals support or follow certain ideologies. These ideologies become the guide or way for the individuals to achieve their long term goals. It is motivated from the competition within the members of a political party as well as competition amongst the leaders of different parties. Every local party follows certain ideology which motivates certain individuals stay associated with that party. Once associated with a party, every individual exhibits a self-interested behaviour and competes with its party members to improve and promote its rank. Every individual looks at its own local party leader as a benchmark and tries to reach as close as possible to it. Also, the individual watches other party leaders and compares itself with that leader. This may motivate it to choose different ideology associated with another party. Furthermore, every local party leader always desires to be a global leader. In other words, it competes with the other party leaders to be a global leader. Moreover, the local party leader desires to remain at least its own party leader. Thus it also competes with the second best in the party which always desires to catch the local party leader position. In addition, the lowest rank individual following the party ideology desires to climb up in the party; however for prolonged time if it understands that following its current party ideology is not improving its rank. Such deserted individual may change the ideology and resort to another party ideology. This competitive behaviour of individuals following certain ideology to improve and climb up in the party as well as compete with other party members is modelled. The mechanism enabled the IA to solve several numerical optimization problems with superior performance in terms of solution quality and computational cost as compared with other existing algorithms.

The remainder parts in this work are organized as follows. Section 2 describes mechanism of IA. Section 3 provides the detail results of the computational experiments conducted to validate the algorithm. In Section 4, conclusions and future directions are provided.

2. Ideology Algorithm (IA)

In the context of the IA, every member or individual associated with a party is a possible solution. Its position in the party depends on the quality or fitness of its solution (objective function value). The individual with the best solution in a party is considered as local party leader and the individual best amongst all the party leaders is considered as the global leader. The local party leader competes with every other party leader with a desire to be a global leader. It also competes with the second best individual in its own party as it is challenged by the second best in the party which desires to be the local party leader. The earlier makes the party leaders explore and locate promising search space. The later forces the party leader to look for a better solution in its own local neighbourhood as well as the second best in the party. This may increase its chances of remaining as the local party leader and improve. The individual in the party with worst solution checks the difference between its own solution and the penultimate worst in the same party. If the difference is greater than a pre-specified value then such deserted individual understands that following the current party ideology is not worth for it. This makes him switchover to another party in a hope to be better off and climb up in that party. The framework makes the individual in every party to directly and indirectly compete with the same party individuals as well as other party individuals. This essentially makes every party to remain in competition and grow which motivates the individuals search for better solutions. The IA procedure is explained below in detail.

Consider a general unconstrained problem (in minimization sense) as follows:

$$\begin{aligned} & \text{Minimize } f(\mathbf{x}) = f(x_1, \dots, x_i, \dots, x_N) \\ & \text{Subject to } \Psi_i^l \leq x_i \leq \Psi_i^u, \quad i = 1, \dots, N \end{aligned} \tag{1}$$

The IA procedure starts with equally dividing the entire population into P parties. In the beginning, the number of individuals M_p in every party p ($p = 1, \dots, P$) is equal, i.e. $M_1 = \dots, M_p, \dots, M_P$. Also the desertion parameter T associated with the worst individual, convergence parameter ε , maximum number of iterations I_{max} and reduction factor $R \in [0, 1]$ are chosen.

Step 1 (Party Formation)

Equally divide the sampling space associated with every variable x_i , $i = 1, \dots, N$ into P party subspaces, i.e. $\forall x_i \Psi_i^p = [\Psi_i^{l,p}, \Psi_i^{u,p}]$, $p, p = 1, \dots, P$.

Step 2 (Evaluation)

From within every party $p = 1, \dots, P$ subspace $\Psi_i^p = [\Psi_i^{l,p}, \Psi_i^{u,p}]$ associated with every variable x_i , $i = 1, \dots, N$, M_p values are randomly sampled and associated objective functions are evaluated. Then the individuals associated with every party p , $p = 1, \dots, P$ could be represented as follows:

$$\forall p \quad f(x_{1,m_p}, \dots, x_{i,m_p}, \dots, x_{N,m_p}), \quad m_p = 1, \dots, M_p \quad (2)$$

or

$$p = \begin{bmatrix} f(x_{1,1}, \dots, x_{i,1}, \dots, x_{N,1}) \\ \vdots \\ f(x_{1,m_p}, \dots, x_{i,m_p}, \dots, x_{N,m_p}) \\ \vdots \\ f(x_{1,M_p}, \dots, x_{i,M_p}, \dots, x_{N,M_p}) \end{bmatrix}, p = 1, \dots, P$$

Step 3 (Local Party Ranking)

From within every party p , $p = 1, \dots, P$ the evaluated individuals are ranked from the best individual referred to as local party leader $L_{p,b}$ to the local worst individual $L_{p,w}$ as shown in Figure 2.1.

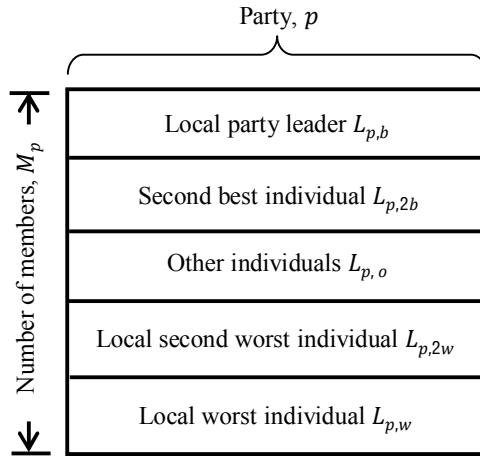


Figure 2.1: The arrangement of individuals in a party.

$$L_{p,b} = \min \left(\begin{bmatrix} f(x_{1,1}, \dots, x_{i,1}, \dots, x_{N,1}) \\ \vdots \\ f(x_{1,m_p}, \dots, x_{i,m_p}, \dots, x_{N,m_p}) \\ \vdots \\ f(x_{1,M_p}, \dots, x_{i,M_p}, \dots, x_{N,M_p}) \end{bmatrix} \right) = f(x_1^{L_{p,b}}, \dots, x_i^{L_{p,b}}, \dots, x_N^{L_{p,b}}) \quad (3)$$

$$L_{p,w} = \max \left(\begin{bmatrix} f(x_{1,1}, \dots, x_{i,1}, \dots, x_{N,1}) \\ \vdots \\ f(x_{1,m_p}, \dots, x_{i,m_p}, \dots, x_{N,m_p}) \\ \vdots \\ f(x_{1,M_p}, \dots, x_{i,M_p}, \dots, x_{N,M_p}) \end{bmatrix} \right) = f(x_1^{L_{p,w}}, \dots, x_i^{L_{p,w}}, \dots, x_N^{L_{p,w}}) \quad (4)$$

The second best individual $L_{p,2b}$ is worse than Local best individual $L_{p,b}$; however, better than every other individual in the party p , i.e. $L_{p,b} < L_{p,2b} < L_{p,o} < L_{p,2w} < L_{p,w}$. The Second worst individual $L_{p,2w}$ is better than the local worst individual $L_{p,w}$; however, worse than every other individual in the party p , i.e. $L_{p,w} > L_{p,2w} > L_{p,o} > L_{p,2b} > L_{p,b}$.

Step 4 (Competition and Improvement for local party leader $L_{p,b}$)

Every local party leader $L_{p,b}$, ($p = 1, \dots, P$) seeks to improve itself through introspection, local competition and global competition. The *introspection* refers to searching the close neighbourhood of its own current solution by modifying the current sampling space associated with its every variable $x_i^{L_{p,b}}$, $i = 1, \dots, N$ as follows:

$$\Psi_{i,insp}^{L_{p,b}} \in [x_i^{L_{p,b}} - R \times (\|\Psi_i^{u,p}\| - \|\Psi_i^{l,p}\|), x_i^{L_{p,b}} + R \times (\|\Psi_i^{u,p}\| - \|\Psi_i^{l,p}\|)] \quad (5)$$

The *local competition* refers to competing with the second best $L_{p,2b}$ in its own party by searching in the close neighbourhood of its current solution. In other words, the current sampling space of every variable i , $i = 1, \dots, N$ associated with every local party leader $L_{p,b}$, ($p = 1, \dots, P$) is updated to the close neighborhood of the local best solution $L_{p,2b}$ as follows:

$$\Psi_{i,lcmpl}^{L_{p,b}} \in [x_i^{L_{p,2b}} - R \times (\|\Psi_i^{u,p}\| - \|\Psi_i^{l,p}\|), x_i^{L_{p,2b}} + R \times (\|\Psi_i^{u,p}\| - \|\Psi_i^{l,p}\|)] \quad (6)$$

Local competition is necessary as the local party leader will always try to remain the best in its own party which may further lead the algorithm to efficiently search for better solution.

The *global competition* refers to searching in the close neighbourhood of the global leader. In other words, the current sampling space of every variable i , $i = 1, \dots, N$ associated with every local party leader $L_{p,b}$ ($p = 1, \dots, P$) is updated to the close neighbourhood of the global best solution $L_{p,gb}$ as follows:

$$\Psi_{i,gcomp}^{L_{p,b}} \in \left[x_i^{L_{p,gb}} - R \times (\|\Psi_i^{u,p}\| - \|\Psi_i^{l,p}\|), x_i^{L_{p,gb}} + R \times (\|\Psi_i^{u,p}\| - \|\Psi_i^{l,p}\|) \right] \quad (7)$$

where $L_{p,gb} = \min(L_{p,b})$, $p = 1, \dots, P$ or $L_{p,gb} = \min(L_{1,b}, \dots, L_{p,b}, \dots, L_{P,b})$

Then, the local party leader $L_{p,b}$ samples variable values from within the updated sampling intervals $\Psi_{i,insp}^{L_{p,b}}$, $\Psi_{i,lcmp}^{L_{p,b}}$ and $\Psi_{i,gcomp}^{L_{p,b}}$ formed using introspection, local competition and global competition, respectively and calculates corresponding objective functions. Then one solution from within the three choices is selected based on the roulette wheel selection approach [43].

For each local worst individual $L_{p,w}$ ($p = 1, 2, \dots, P$), the distance d between itself and the second worst individual $L_{p,2w}$ is evaluated as follows:

$$d = \|L_{p,w} - L_{p,2w}\| \quad (8)$$

If the difference is higher than a pre-specified value T , then the individual understands that it is deserted (worst off) and switches over to another randomly selected party $\hat{p} \in [1, \dots, P]$, i.e.

$$\begin{aligned} L_{p,w}^p &= L_{\hat{p},w+1}^{\hat{p}} && \text{if } d \geq T \\ L_{p,w}^p &= L_{p,o}^p && \text{otherwise} \end{aligned} \quad (9)$$

where $L_{p,o}$ ($o = 1, 2, \dots, O$) is referred to as ordinary individual for party ($p = 1, 2, \dots, P$) and $M_p - 2 \leq O < M_p$.

Step 5(Updating party individuals)

Every ordinary individual $L_{p,o}$ ($o = 1, 2, \dots, O$) for every party ($p = 1, 2, \dots, P$) searches one solution in its own neighbourhood $\Psi_{i,o}^{L_{p,o}}$, every local party leader's neighborhood $\Psi_{i,lopl}^{L_{p,o}}$, ($p = 1, 2, \dots, P$). The best O solutions are chosen from within this pool and the party individuals are updated. In other words, the current sampling space of every

variable (i , $i = 1, \dots, N$) associated with every individual $L_{p,o}$ ($o = 1, 2, \dots, O$), ($p = 1, \dots, P$) other than the local party leader $L_{p,b}$ and deserted individual $L_{p,w}$ updates its sampling space in the close neighbourhood of itself (refer to Equation (10)), and every local party leader $L_{p,b}$ ($p = 1, \dots, P$) as follows:

$$\Psi_{i,o}^{L_{p,o}} \in [x_i^{L_{p,o}} - R \times (\|\Psi_i^{u,p}\| - \|\Psi_i^{l,p}\|), x_i^{L_{p,o}} + R \times (\|\Psi_i^{u,p}\| - \|\Psi_i^{l,p}\|)] \quad (10)$$

$$\begin{aligned} \Psi_{i,lopl}^{L_{p,o}} &\in [x_i^{L_{p,b}} - R \times (\|\Psi_i^{u,p}\| - \|\Psi_i^{l,p}\|), x_i^{L_{p,b}} + R \times (\|\Psi_i^{u,p}\| - \|\Psi_i^{l,p}\|)], \\ &(p = 1, \dots, P) \end{aligned} \quad (11)$$

In this way, each individual of every party is updated.

Step 6 (Convergence)

The parties are considered converged if any of the following conditions satisfied, else continue to Step 1:

- a) There is no significant improvement in the local party leader solutions for a significant number of iterations and/or
- b) The maximum number of iterations I_{max} is reached.

It is important to mention that the number of party members may change in every iteration as some of the individuals may leave a party and join any other in hope to improve. Figure 2.2 shows the general structure of IA, where Figure 2.3 shows the flow chart of IA. The working mechanism of the IA is illustrated in Figure 2.4, where the dots represent individuals or party members and the centralized dot represents leader in a party. Figure 2.5 illustrates the movement of individuals during a run solving multimodal Ackley function. It exhibits the ability of the algorithm to quickly jump out of the local minima and reach the global minimum solution.

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- 1: Initialization
 - 2: Party formation
 - 3: Generation of party individuals
 - 4: **repeat**
 - 5: Evaluation
 - 6: Local ranking
 - 7: Competition and improvement
 - 8: Updating party individuals
 - 9: **until** convergence
-

Figure 2.2: General structure of IA

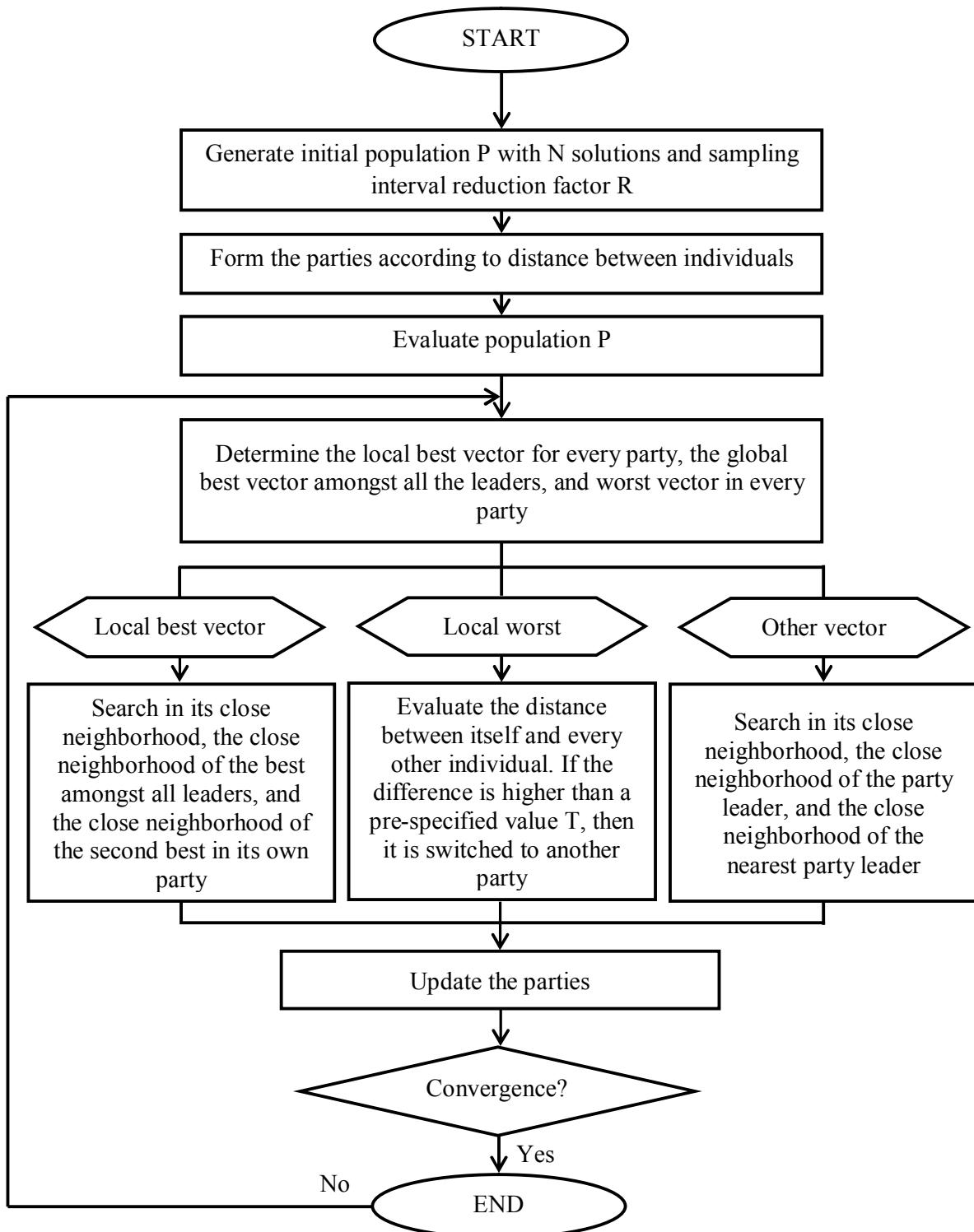
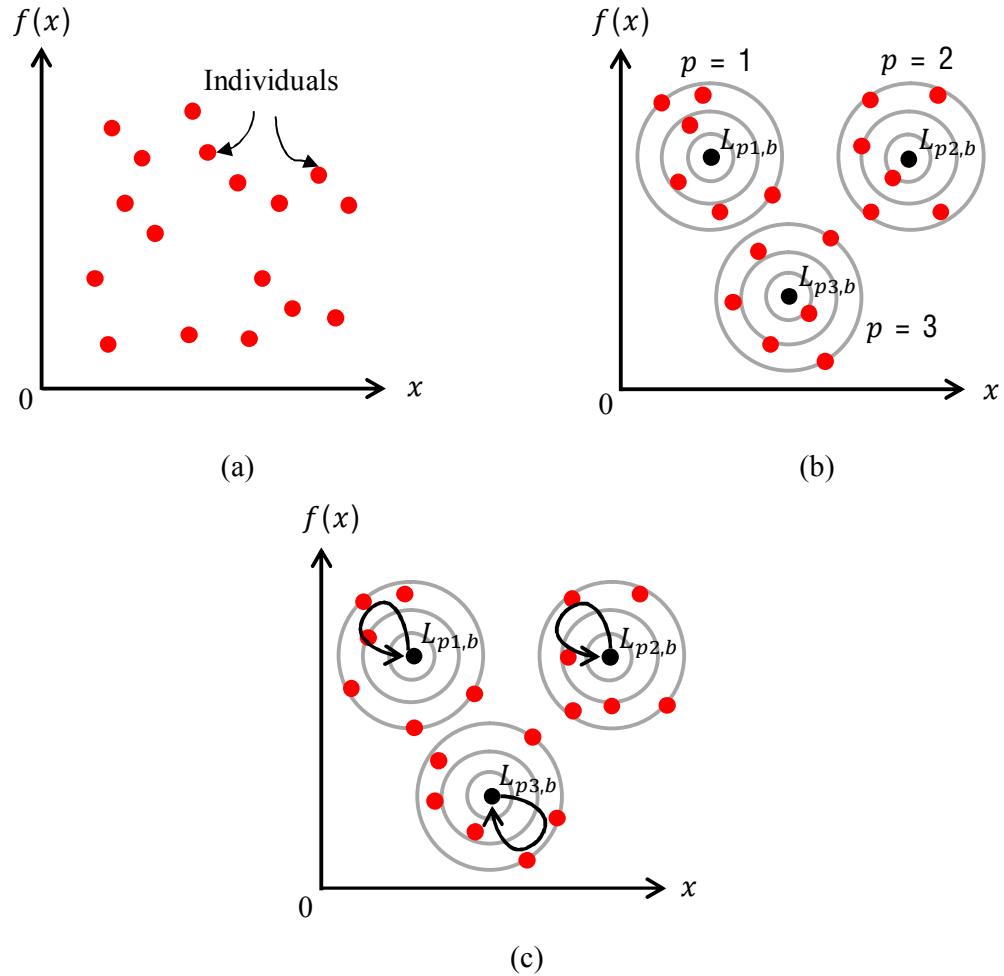


Figure 2.3: Flow chart of Ideology Algorithm (IA)



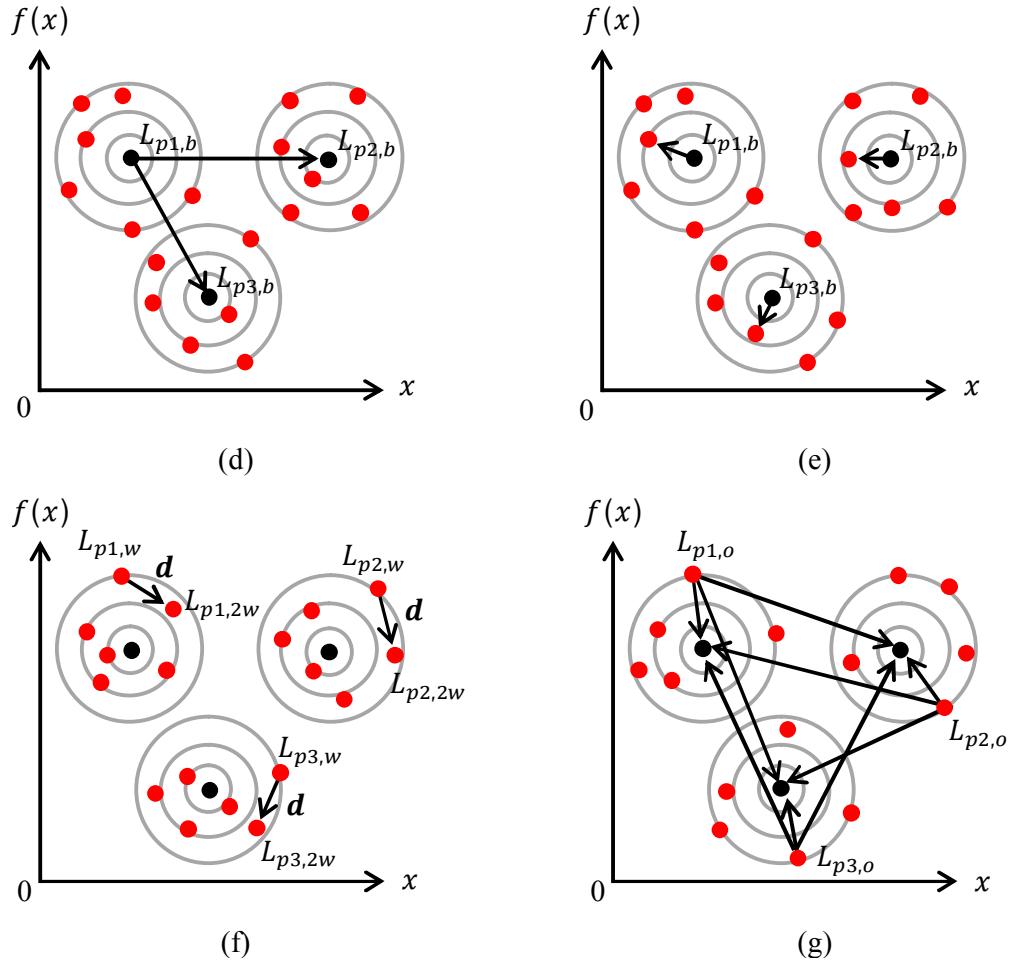
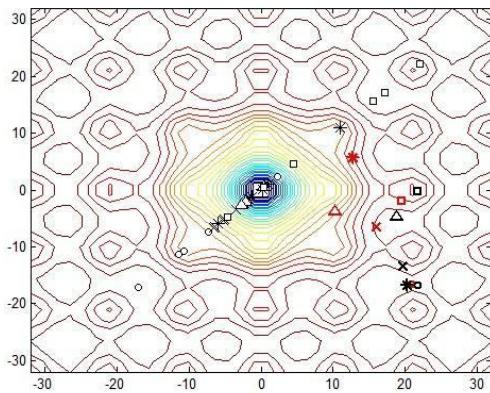
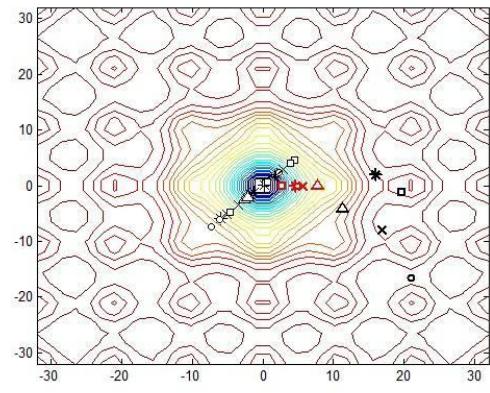


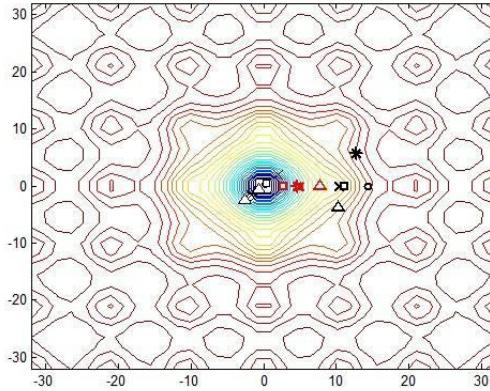
Figure 2.4: (a) Initialisation of the individuals; (b) Associate individuals to p parties ($p = 1, 2, \dots, P$); (c) Introspection; (d) Global competition; (e) Local competition; (f) Search by local worst vector; (g) Search by ordinary individual $L_{p,o}$



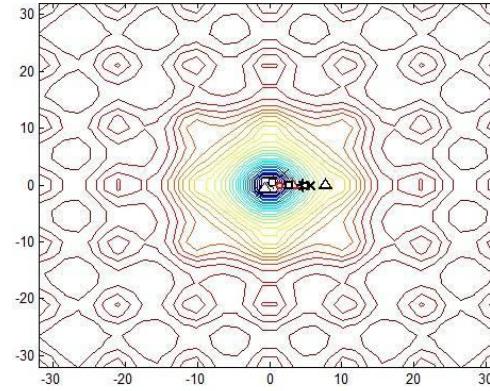
(a)



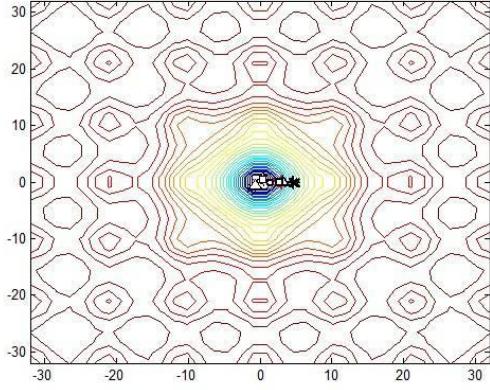
(b)



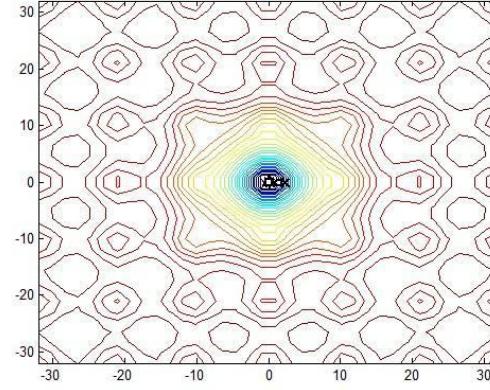
(c)



(d)



(e)



(f)

Figure 2.5: Search pattern for benchmark function F5 (Ackley). (a) Initialization of population, (b) Iteration 1, (c) Iteration 3, (d) Iteration 5, (e) Iteration 10, (f) Convergence

3. Results and Discussion

In this section, the tests and benchmark problems, statistical analysis, control parameters and convergence conditions used for the IA in the tests are presented. The performance of IA is investigated in detail. For experiments, each algorithm (PSO, CMAES, ABC, JDE, CLPSO, SADE, BSA and IA) is coded in MATLAB R2013a on Windows Platform with a T6400@4GHz Intel Core 2 Duo processor with 4GB RAM.

3.1 Benchmark Problems

Two tests are conducted to examine the performance of IA and the comparison algorithms in solving the numerical optimization problems. Test 1 involved 50 widely used benchmark problems [15, 42]. Table 3.1 summarizes several features of the benchmark problems used in Test 1. Test 2 involved 25 benchmark problems used in CEC2005 [46]. Table 3.2 summarizes several features of the benchmark problems used in Test 2.

Table 3.1: The benchmark problems used in Test 1 (Dim = Dimension; Low and Up = Limitations of search space; U = Unimodal; M = Multimodal; S = Separable; N = Non-separable)

Problem	Name	Type	Low	Up	Dimension
F1	Foxholes	MS	-65.536	65.536	2
F2	Goldstein-Price	MN	-2	2	2
F3	Penalized	MN	-50	50	30
F4	Penalized2	MN	-50	50	30
F5	Ackley	MN	-32	32	30
F6	Beale	UN	-4.5	4.5	5
F7	Bohachecky1	MS	-100	100	2
F8	Bohachecky2	MN	-100	100	2
F9	Bohachecky3	MN	-100	100	2
F10	Booth	MS	-10	10	2
F11	Branin	MS	-5	10	2
F12	Colville	UN	-10	10	4
F13	Dixon-Price	UN	-10	10	30
F14	Easom	UN	-100	100	2
F15	Fletcher	MN	-3.1416	3.1416	2
F16	Fletcher	MN	-3.1416	3.1416	5
F17	Fletcher	MN	-3.1416	3.1416	10
F18	Griewank	MN	-600	600	30
F19	Hartman3	MN	0	1	3
F20	Hartman6	MN	0	1	6
F21	Kowalik	MN	-5	5	4
F22	Langermann2	MN	0	10	2
F23	Langermann5	MN	0	10	5
F24	Langermann10	MN	0	10	10
F25	Matyas	UN	-10	10	2
F26	Michalewics2	MS	0	3.1416	2
F27	Michalewics5	MS	0	3.1416	5
F28	Michalewics10	MS	0	3.1416	10
F29	Perm	MN	-4	4	4
F30	Powell	UN	-4	5	24
F31	Powersum	MN	0	4	4
F32	Quartic	US	-1.28	1.28	30
F33	Rastrigin	MS	-5.12	5.12	30
F34	Rosenbrock	UN	-30	30	30
F35	Schaffer	MN	-100	100	2
F36	Schwefel	MS	-500	500	30
F37	Schwefel_1_2	UN	-100	100	30
F38	Schwefel_2_22	UN	-10	10	30
F39	Shekel10	MN	0	10	4
F40	Shekel5	MN	0	10	4
F41	Shekel7	MN	0	10	4
F42	Shubert	MN	-10	10	2
F43	Six-hump camelback	MN	-5	5	2
F44	Sphere2	US	-100	100	30
F45	Step2	US	-100	100	30
F46	Stepint	US	-5.12	5.12	5
F47	Sumsquares	US	-10	10	30
F48	Trid6	UN	-36	36	6
F49	Trid10	UN	-100	100	10
F50	Zakharov	UN	-5	10	10

Table 3.2: The benchmark problems used in Test 2 (Dim = Dimension; Low and Up = Limitations of search space; U = Unimodal; M = Multimodal; E = Expanded; H = Hybrid)

Problem	Name	Type	Low	Up	Dimension
F51	Shifted sphere	U	-100	100	10
F52	Shifted Schwefel	U	-100	100	10
F53	Shifted rotated high conditioned elliptic function	U	-100	100	10
F54	Shifted Schwefels problem 1.2 with noise	U	-100	100	10
F55	Schwefels problem 2.6	U	-100	100	10
F56	Shifted Rosenbrock's	M	-100	100	10
F57	Shifted rotated Griewank's	M	0	600	10
F58	Shifted rotated Ackley's	M	-32	32	10
F59	Shifted Rastrigin's	M	-5	5	10
F60	Shifted rotated Rastrigin's	M	-5	5	10
F61	Shifted rotated Weierstrass	M	-0.5	0.5	10
F62	Schwefels problem 2.13	M	-100	100	10
F63	Expanded extended Griewank's + Rosenbrock's	E	-3	1	10
F64	Expanded rotated extended Scaffes	E	-100	100	10
F65	Hybrid composition function	HC	-5	5	10
F66	Rotated hybrid comp. Fn 1	HC	-5	5	10
F67	Rotated hybrid comp. Fn 1 with noise	HC	-5	5	10
F68	Rotated hybrid comp. Fn 2	HC	-5	5	10
F69	Rotated hybrid comp. Fn 2 with narrow global optimal	HC	-5	5	10
F70	Rotated hybrid comp. Fn 2 with the global optimum	HC	-5	5	10
F71	Rotated hybrid comp. Fn 3	HC	-5	5	10
F72	Rotated hybrid comp. Fn 3 with high condition number matrix	HC	-5	5	10
F73	Non-continuous rotated hybrid comp. Fn 3	HC	-5	5	10
F74	Rotated hybrid comp. Fn 4	HC	-5	5	10
F75	Rotated hybrid comp. Fn 4	HC	-2	5	10

3.2 Control Parameters

The values of the control parameters used in the experiments for IA are listed as shown in Table 3.3.

Table 3.3: The relevant control parameters used in the experiments for IA

Control Parameter	Numerical Values
Maximum number of iteration (I_{max})	30
Number of parties presented (p)	5
Initial Population size (x)	150
Random vector (R)	10000

3.3 Stopping Criterion

The predetermined stopping criterion is set to terminate the algorithms.

- Stop when the absolute value of the objective function evaluations is less than 10^{-16} .
- Stop when the maximum number of function evaluations reaches 200000.
- Stop when the maximum number of iterations (I_{max}) is reached.

Parametric tests have been commonly used in the analysis of experiments. For example, a common way to test whether the difference between the results of two algorithms is non-random is to apply a paired t-test, which checks whether the average difference in their performance over the problems is significantly different from zero. Non-parametric tests,

besides their original definition for dealing with nominal or ordinal data, can be also applied to continuous data by conducting ranking-based transformations, adjusting the input data to the test requirements. They can perform two classes of analysis: pairwise comparisons and multiple comparisons. Pairwise statistical procedures perform individual comparisons between two algorithms, obtaining in each application a p-value independent from another one [41]. Pairwise comparisons are the simplest kind of statistical tests that a researcher can apply within the framework of an experimental study. Such tests are directed to compare the performance of two algorithms when applied to a common set of problems. In multi-problem analysis, a value for each pair of algorithm is required (often an average value from several runs). In this section, we focus on the Sign test, which is a quick and easy procedure that can provide a clearer view about the comparison. Then, the Wilcoxon signed ranks test is introduced as an example of a simple non-parametric test for pairwise statistical comparisons.

3.4 Statistical Analysis

The Wilcoxon signed ranks test is a non-parametric procedure employed in hypothesis testing situations, involving a design with two samples. It is commonly used for answering the following question: do two samples represent two different populations? This is analogous to the paired t-test in non-parametric statistical procedures. Thus, it is a pairwise test that aims to detect significant differences between two sample means, i.e., the behaviour of two algorithms.

Table 3.4 shows the mean runtimes and simple statistical values for the results obtained in Test 1, whereas Table 3.6 lists the algorithms that obtained statistically better solutions compared with the other algorithms in Test 1, based on the Wilcoxon Signed-Rank Test. Table 3.5 shows the mean runtimes and simple statistical values for the results obtained in Test 2, whereas Table 3.7 lists the algorithms that provided statistically better solutions compared with the other algorithms in Test 2, based on the Wilcoxon Signed-Rank Test.

Table 3.8 presents the multi-problem-based pairwise statistical comparison results using the averages of the global minimum values obtained through 30 runs of IA and the comparison algorithms to solve the benchmark problems in Tests 1 and Test 2. The results indicate that IA was statistically more successful than most of the comparison algorithms with a statistical significance value $\alpha = 0.05$.

In Tables 3.6 and 3.7, a ‘+’ sign indicates cases in which the null hypothesis is rejected and IA displays a statistically superior performance in the problem-based statistical comparison tests at the 95% significance level ($\alpha = 0.05$). The ‘–’ sign indicates cases in which the null hypothesis was rejected and IA displayed an inferior performance; and ‘=’ indicates cases in which there was no statistical difference between the two algorithms’ success in solving the problems. The last rows of Tables 3.6 and 3.7 depict the total counts in a format of ‘+ / = / –’ for the three statistical significance cases (marked with ‘+’, ‘=’ or ‘–’) in the pairwise comparison.

When the (+ / = / –) values are examined, it can be said that IA is statistically more successful than most of the other algorithms in solving the problems in Tests 1 and 2.

Although in Figure 3.8, the successes IA and BSA have had are statistically identical; IA has provided statistically better solutions than other algorithms.

Table 3.4: Statistical solutions obtained by PSO, CMAES, ABC, CLPSO, SADE, BSA and proposed IA in Test 1 (Mean = Mean solution; Std. Dev. = Standard-deviation of mean solution; Best = Best solution; Runtime = Mean runtime in seconds)

Problem	Statistics	PSO2011	CMAES	ABC	JDE	CLPSO	SADE	BSA	IA
F1	Mean	1.3316029264876300	10.0748846367972000	0.9980038377944500	1.0641405484285200	1.8209961275956800	0.9980038377944500	0.9980038377944500	0.9980038690000000
	Std. Dev.	0.9455237994690700	8.0277365400340800	0.0000000000000001	0.3622456829347420	1.6979175079427900	0.0000000000000000	0.0000000000000000	0.000000000000035
	Best	0.9980038377944500	0.9980038377944500	0.9980038377944500	0.9980038377944500	0.9980038377944500	0.9980038377944500	0.9980038377944500	0.9980038685998520
	Runtime	72.527	44.788	64.976	51.101	61.650	66.633	38.125	43.535
F2	Mean	2.9999999999999200	21.8999999999995000	3.0000000465423000	2.9999999999999200	3.000000000000700	2.9999999999999200	2.9999999999999200	3.0240147900000000
	Std. Dev.	0.000000000000013	32.6088098948516000	0.0000002350442161	0.000000000000013	0.000000000007941	0.000000000000020	0.000000000000011	0.0787814840000000
	Best	2.9999999999999200	2.9999999999999200	2.9999999999999200	2.9999999999999200	2.9999999999999200	2.9999999999999200	2.9999999999999200	3.0029461118668700
	Runtime	17.892	24.361	16.624	7.224	24.784	28.699	7.692	41.343
F3	Mean	0.1278728062391630	0.0241892995662904	0.0000000000000004	0.0034556340083499	0.0000000000000000	0.0034556340083499	0.0000000000000000	0.3536752140000000
	Std. Dev.	0.2772792346028400	0.0802240262581864	0.0000000000000001	0.0189272869685522	0.0000000000000000	0.0189272869685522	0.0000000000000000	1.4205454130000000
	Best	0.0000000000000000	0.0000000000000000	0.0000000000000003	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0014898619035614
	Runtime	139.555	5.851	84.416	9.492	38.484	15.992	18.922	34.494
F4	Mean	0.0043949463343535	0.0003662455278628	0.0000000000000004	0.0007324910557256	0.0000000000000000	0.0440448539086004	0.0000000000000000	0.0179485820000000
	Std. Dev.	0.0054747064090174	0.0020060093719584	0.0000000000000001	0.0027875840585535	0.0000000000000000	0.2227372747439610	0.0000000000000000	0.0526650620000000
	Best	0.0000000000000000	0.0000000000000000	0.0000000000000003	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000165491
	Runtime	126.507	6.158	113.937	14.367	48.667	33.019	24.309	322.808
F5	Mean	1.5214322973725000	11.7040011684582000	0.000000000000340	0.0811017056422860	0.1863456353861950	0.7915368220335460	0.0000000000000105	0.0000000000000009
	Std. Dev.	0.6617570384662600	9.7201961540865200	0.000000000000035	0.3176012689149320	0.4389839299322230	0.7561593402959740	0.0000000000000034	0.0000000000000000
	Best	0.0000000000000080	0.0000000000000080	0.0000000000000293	0.0000000000000044	0.0000000000000080	0.0000000000000044	0.0000000000000080	0.0000000000000009
	Runtime	63.039	3.144	23.293	11.016	45.734	40.914	14.396	49.458
F6	Mean	0.0000000041922968	0.2540232169641050	0.0000000000000028	0.0000000000000000	0.000044354499943	0.0000000000000000	0.0000000000000000	0.0082236060000000
	Std. Dev.	0.0000000139615552	0.3653844307786430	0.0000000000000030	0.0000000000000000	0.0001015919507724	0.0000000000000000	0.0000000000000000	0.0000000000000000
	Best	0.0000000000000000	0.0000000000000000	0.0000000000000005	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0082236059357692
	Runtime	32.409	4.455	22.367	1.279	125.839	4.544	0.962	50.246
F7	Mean	0.0000000000000000	0.0622354533647150	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000
	Std. Dev.	0.0000000000000000	0.1345061339146580	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000
	Best	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000
	Runtime	16.956	6.845	1.832	1.141	2.926	4.409	0.825	38.506
F8	Mean	0.0000000000000000	0.0072771062590204	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000
	Std. Dev.	0.0000000000000000	0.0398583525142753	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000
	Best	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000

	Runtime	17.039	2.174	1.804	1.139	2.891	4.417	0.824	39.023
F9	Mean	0.0000000000000000	0.0001048363065820	0.0000000000000006	0.0000000000000000	0.0000193464326398	0.0000000000000000	0.0000000000000000	0.0000000000000000
	Std. Dev.	0.0000000000000000	0.0005742120996051	0.0000000000000003	0.0000000000000000	0.0000846531630676	0.0000000000000000	0.0000000000000000	0.0000000000000000
	Best	0.0000000000000000	0.0000000000000000	0.0000000000000001	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000
	Runtime	17.136	2.127	21.713	1.129	33.307	4.303	0.829	40.896
F10	Mean	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0006005122443674	0.0000000000000000	0.0000000000000000	0.8346587090000000
	Std. Dev.	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0029861918862801	0.0000000000000000	0.0000000000000000	0.0000000000000005
	Best	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.8346587086917530
	Runtime	17.072	1.375	22.395	1.099	28.508	4.371	0.790	39.978
F11	Mean	0.3978873577297380	0.6372170283279430	0.3978873577297380	0.3978873577297380	0.3978873577297380	0.3978873577297380	0.3978873577297380	0.4156431270000000
	Std. Dev.	0.0000000000000000	0.7302632173480510	0.0000000000000000	0.0000000000000000	0.0000000000000049	0.0000000000000000	0.0000000000000000	0.0406451050000000
	Best	0.3978873577297380	0.3978873577297380	0.3978873577297380	0.3978873577297380	0.3978873577297380	0.3978873577297380	0.3978873577297380	0.4012748152492080
	Runtime	17.049	24.643	10.941	6.814	17.283	27.981	5.450	40.099
F12	Mean	0.0000000000000000	0.0000000000000000	0.0715675060725970	0.0000000000000000	0.1593872502094070	0.0000000000000000	0.0000000000000000	0.0014898620000000
	Std. Dev.	0.0000000000000000	0.0000000000000000	0.0579425013417103	0.0000000000000000	0.6678482786713720	0.0000000000000000	0.0000000000000000	0.0000000000000000
	Best	0.0000000000000000	0.0000000000000000	0.0013425253994745	0.0000000000000000	0.00000094069599934	0.0000000000000000	0.0000000000000000	0.0082029783984983
	Runtime	44.065	1.548	21.487	1.251	166.965	4.405	2.460	48.067
F13	Mean	0.6666666666666750	0.6666666666666670	0.000000000000038	0.6666666666666670	0.0023282133668190	0.6666666666666670	0.6444444444444440	0.2528116640000000
	Std. Dev.	0.0000000000000022	0.0000000000000000	0.0000000000000012	0.0000000000000002	0.0051792840882291	0.0000000000000000	0.1217161238900370	0.0000000006509080
	Best	0.6666666666666720	0.6666666666666670	0.0000000000000021	0.6666666666666670	0.00000120708732167	0.6666666666666670	0.0000000000000000	0.2528116633611470
	Runtime	167.094	3.719	37.604	18.689	216.261	47.833	21.192	67.463
F14	Mean	-1.0000000000000000	-0.1000000000000000	-1.0000000000000000	-1.0000000000000000	-1.0000000000000000	-1.0000000000000000	-1.0000000000000000	-0.9997989620000000
	Std. Dev.	0.0000000000000000	0.3051285766293650	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000167151
	Best	-1.0000000000000000	-1.0000000000000000	-1.0000000000000000	-1.0000000000000000	-1.0000000000000000	-1.0000000000000000	-1.0000000000000000	-0.9997989624626810
	Runtime	16.633	3.606	13.629	6.918	16.910	28.739	5.451	39.685
F15	Mean	0.0000000000000000	1028.3930784026900000	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000
	Std. Dev.	0.0000000000000000	1298.1521820113500000	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000
	Best	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000
	Runtime	27.859	15.541	40.030	2.852	4.030	6.020	2.067	38.867
F16	Mean	48.7465164446927000	1680.3460230073400000	0.0218688498331872	0.9443728655432830	81.7751618148164000	0.0000000000000000	0.0000000000000000	0.0000000000000000
	Std. Dev.	88.8658510972991000	2447.7484859066000000	0.0418409568792831	2.8815514827061600	379.9241117377270000	0.0000000000000000	0.0000000000000000	0.0000000000000000
	Best	0.0000000000000000	0.0000000000000000	0.0000000000000016	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000
	Runtime	95.352	11.947	44.572	4.719	162.941	5.763	7.781	48.262

F17	Mean	918.9518492782850000	12340.2283326398000000	11.0681496253548000	713.7226974626920000	0.8530843976878610	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000	
	Std. Dev.	1652.4810858411400000	22367.1698875802000000	9.8810950146557100	1710.071307430120000	2.9208253191698800	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000	
	Best	0.0000000000000000	0.0000000000000000	0.3274654777056860	0.0000000000000000	0.0016957837829822	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000	
	Runtime	271.222	7.631	43.329	16.105	268.894	168.310	33.044	69.060		
F18	Mean	0.0068943694819713	0.0011498935321349	0.0000000000000000	0.0048193578543185	0.0000000000000000	0.0226359326967139	0.0004930693556077	0.0000000000000000		
	Std. Dev.	0.0080565201649587	0.0036449413521107	0.0000000000000001	0.0133238235582874	0.0000000000000000	0.0283874287215679	0.0018764355751644	0.0000000000000000		
	Best	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000		
	Runtime	73.895	2.647	19.073	6.914	14.864	25.858	5.753	2.717		
F19	Mean	-3.8627821478207500	-3.7243887744664700	-3.8627821478207500	-3.8627821478207500	-3.8627821478207500	-3.8627821478207500	-3.8627821478207500	-3.8627821478207500	-3.8596352620000000	
	Std. Dev.	0.0000000000000027	0.5407823545193820	0.0000000000000024	0.0000000000000027	0.0000000000000027	0.0000000000000027	0.0000000000000027	0.0000000000000027	0.0033967610000000	
	Best	-3.8627821478207600	-3.8627821478207600	-3.8627821478207600	-3.8627821478207600	-3.8627821478207600	-3.8627821478207600	-3.8627821478207600	-3.8627821478207600	-3.8613076574052300	
	Runtime	19.280	21.881	12.613	7.509	17.504	24.804	6.009	46.167		
F20	Mean	-3.3180320675402500	-3.2942534432762600	-3.3219951715842400	-3.2982165473202600	-3.3219951715842400	-3.3140689634962500	-3.3219951715842400	-2.5710247593206100		
	Std. Dev.	0.0217068148263721	0.0511458075926848	0.0000000000000014	0.0483702518391572	0.0000000000000013	0.0301641516823498	0.0000000000000013	0.0000000000000009		
	Best	-3.3219951715842400	-3.3219951715842400	-3.3219951715842400	-3.3219951715842400	-3.3219951715842400	-3.3219951715842400	-3.3219951715842400	-2.5710247593206100		
	Runtime	26.209	7.333	13.562	8.008	20.099	33.719	6.822	59.083		
F21	Mean	0.0003074859878056	0.0064830287538208	0.0004414866359626	0.0003685318137604	0.0003100479704151	0.0003074859878056	0.0003074859878056	0.0016993410000000		
	Std. Dev.	0.0000000000000000	0.0148565973286009	0.0000568392289725	0.0002323173367683	0.0000059843325073	0.0000000000000000	0.0000000000000000	0.0000013058400000		
	Best	0.0003074859878056	0.0003074859878056	0.0003230956007045	0.0003074859878056	0.0003074859941292	0.0003074859878056	0.0003074859878056	0.001698914552560		
	Runtime	84.471	13.864	20.255	7.806	156.095	45.443	11.722	48.920		
F22	Mean	-1.0809384421344400	-0.7323679641701760	-1.0809384421344400	-1.0764280762657400	-1.0202940450426400	-1.0809384421344400	-1.0809384421344400	-1.4315374190000000		
	Std. Dev.	0.0000000000000006	0.4136688304155380	0.0000000000000008	0.0247042912888477	0.1190811583120530	0.0000000000000005	0.0000000000000005	0.0000000000000009		
	Best	-1.0809384421344400	-1.0809384421344400	-1.0809384421344400	-1.0809384421344400	-1.0809384421344400	-1.0809384421344400	-1.0809384421344400	-1.4315374193830000		
	Runtime	27.372	32.311	27.546	19.673	52.853	36.659	21.421	34.714		
F23	Mean	-1.3891992200744600	-0.5235864386288060	-1.4999990070800800	-1.3431399432579700	-1.4765972735526500	-1.4999992233525000	-1.4821658762555300	-1.5000000000000000		
	Std. Dev.	0.2257194403158630	0.2585330714077300	0.0000008440502079	0.2680292304904580	0.1281777579497830	0.0000000000000009	0.0976772648082733	0.0000000000000000		
	Best	-1.4999992233524900	-0.7977041047646610	-1.4999992233524900	-1.4999992233524900	-1.4999992233524900	-1.4999992233524900	-1.4999992233524900	-1.5000000000000000		
	Runtime	33.809	17.940	37.986	20.333	42.488	36.037	18.930	41.848		
F24	Mean	-0.9166206788680230	-0.3105071678265780	-0.8406348096500680	-0.8827152798835760	-0.9431432797743700	-1.2765515661973800	-1.3127183561646500	-1.5000000000000000		
	Std. Dev.	0.3917752367440500	0.2080317241440800	0.2000966365984320	0.3882445165494030	0.3184175870987750	0.3599594108130040	0.3158807699946290	0.0000000000000000		
	Best	-1.50000000000003800	-0.7976938356122860	-1.4999926800631400	-1.50000000000003800	-1.50000000000003800	-1.50000000000003800	-1.50000000000003800	-1.50000000000003800		
	Runtime	110.798	8.835	38.470	21.599	124.609	47.171	35.358	54.651		
F25	Mean	0.0000000000000000	0.0000000000000000	0.0000000000000004	0.0000000000000000	0.00000041787372626	0.0000000000000000	0.0000000000000000	0.0000000000000000		

	Std. Dev.	0.0000000000000000	0.0000000000000000	0.0000000000000003	0.0000000000000000	0.0000161643637543	0.0000000000000000	0.0000000000000000	0.0000000000000000
	Best	0.0000000000000000	0.0000000000000000	0.0000000000000001	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000
	Runtime	25.358	1.340	19.689	1.142	31.632	4.090	0.813	35.662
F26	Mean	-1.8210436836776800	-1.7829268228561700	-1.8210436836776800	-1.8210436836776800	-1.8210436836776800	-1.8210436836776800	-1.8210436836776800	-1.8203821100000000
	Std. Dev.	0.0000000000000009	0.1450583631808370	0.0000000000000009	0.0000000000000009	0.0000000000000009	0.0000000000000009	0.0000000000000009	0.0000000000000014
	Best	-1.8210436836776800	-1.8210436836776800	-1.8210436836776800	-1.8210436836776800	-1.8210436836776800	-1.8210436836776800	-1.8210436836776800	-1.8203821095139300
	Runtime	19.154	26.249	17.228	9.663	18.091	28.453	7.472	34.891
F27	Mean	-4.6565646397053900	-4.1008953007033700	-4.6934684519571100	-4.6893456932617100	-4.6920941990586400	-4.6884965299983800	-4.6934684519571100	-3.2820108350000000
	Std. Dev.	0.0557021530063238	0.4951250481844850	0.0000000000000009	0.0125797149251589	0.0075270931220834	0.0272323381095561	0.0000000000000008	0.0000000000000023
	Best	-4.6934684519571100	-4.6934684519571100	-4.6934684519571100	-4.6934684519571100	-4.6934684519571100	-4.6934684519571100	-4.6934684519571100	-3.2820108345268900
	Runtime	38.651	10.956	17.663	14.915	25.843	38.446	11.971	45.085
F28	Mean	-8.9717330307549300	-7.6193507368464700	-9.6601517156413500	-9.6397230986132500	-9.6400278592589600	-9.6572038232921700	-9.6601517156413500	-6.2086254390000000
	Std. Dev.	0.4927013165009220	0.7904830398850970	0.0000000000000008	0.0393668145094111	0.0437935551332868	0.0105890022905617	0.0000000000000007	0.0000000000000027
	Best	-9.5777818097208200	-9.1383975057875100	-9.6601517156413500	-9.6601517156413500	-9.6601517156413500	-9.6601517156413500	-9.6601517156413500	-6.2086254392105500
	Runtime	144.093	6.959	27.051	20.803	32.801	46.395	22.250	71.652
F29	Mean	0.0119687224560441	0.0788734736114700	0.0838440014038032	0.0154105130055856	0.0198686590210374	0.0140272066690658	0.0007283694780796	1.3116221610000000
	Std. Dev.	0.0385628598040034	0.1426911799629180	0.0778327303965192	0.0308963906374663	0.0613698943155661	0.0328868042987376	0.0014793717464195	0.5590904820000000
	Best	0.00000044608370213	0.0000000000000000	0.0129834451730589	0.0000000000000000	0.0000175219764526	0.0000000000000000	0.0000000000000000	1.0960146962658900
	Runtime	359.039	17.056	60.216	35.044	316.817	92.412	191.881	34.697
F30	Mean	0.0000130718912008	0.0000000000000000	0.0002604330013462	0.0000000000000001	0.0458769685199585	0.0000002733806735	0.0000000028443186	0.0000000000000000
	Std. Dev.	0.0000014288348929	0.0000000000000000	0.0000394921919294	0.0000000000000002	0.0620254411839524	0.0000001788830279	0.000000003308990	0.0000000000000000
	Best	0.00000095067504097	0.0000000000000000	0.0001682411286088	0.0000000000000000	0.0005277712020642	0.000000094121661	0.000000004769768	0.0000000000000000
	Runtime	567.704	14.535	215.722	194.117	252.779	360.380	144.784	153.221
F31	Mean	0.0001254882834238	0.0000000000000000	0.0077905311094958	0.0020185116261490	0.0002674563703837	0.0000000000000000	0.00000000111676630	0.0071082040000000
	Std. Dev.	0.0001503556280087	0.0000000000000000	0.0062425841086448	0.0077448684015362	0.0003044909265796	0.0000000000000000	0.0000000184322163	0.0000000000000000
	Best	0.0000000156460198	0.0000000000000000	0.0003958766023752	0.0000000000000000	0.00000023064754605	0.0000000000000000	0.0000000000000000	0.0071082039505830
	Runtime	250.248	12.062	34.665	48.692	227.817	220.886	149.882	43.098
F32	Mean	0.0003548345513179	0.0701619169853449	0.0250163252527030	0.0013010316180679	0.0019635752485802	0.0016730768406953	0.0019955316015528	0.0002254250000000
	Std. Dev.	0.0001410817500914	0.0288760292572957	0.0077209314806873	0.0009952078711752	0.004323828633839	0.0007330246909835	0.0009698942217908	0.0005270410000000
	Best	0.0001014332605364	0.029918071536354	0.0094647580732654	0.0001787238105452	0.0004206447422138	0.0005630852254632	0.0006084880639553	0.0000023800831017
	Runtime	290.669	2.154	34.982	82.124	103.283	171.637	48.237	218.722
F33	Mean	25.6367602258676000	95.9799861204982000	0.0000000000000000	1.1276202647057400	0.6301407361590880	0.8622978494808570	0.0000000000000000	0.0000000000000000
	Std. Dev.	8.2943512684216700	56.6919245985100000	0.0000000000000000	1.0688393637536800	0.8046401822326410	0.9323785263847000	0.0000000000000000	0.0000000000000000
	Best	12.9344677422129000	29.8487565993415000	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000

	Runtime	76.083	2.740	4.090	7.635	18.429	23.594	5.401	2.266
F34	Mean	2.6757043114269700	0.3986623855035210	0.2856833465904130	1.063099694802500	5.7631786582751800	1.2137377447007000	0.3986623854300930	0.0000154715000000
	Std. Dev.	12.3490058210004000	1.2164328621946200	0.6247370987465170	1.7930895051734300	13.9484817304201000	1.8518519388285700	1.2164328622195200	0.0000022373400000
	Best	0.0042535368984501	0.0000000000000000	0.0004266049929880	0.0000000000000000	0.0268003205820685	0.0001448955835246	0.0000000000000000	0.0000118803557196
	Runtime	559.966	9.462	35.865	23.278	187.894	268.449	34.681	7.250
F35	Mean	0.0000000000000000	0.4651202457398910	0.0000000000000000	0.0038863639514140	0.0019431819755029	0.0006477273251676	0.0000000000000000	0.0000000000000000
	Std. Dev.	0.0000000000000000	0.0933685176073728	0.0000000000000000	0.0048411743884718	0.0039528023354469	0.0024650053428137	0.0000000000000000	0.0000000000000000
	Best	0.0000000000000000	0.009715909775144	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000
	Runtime	18.163	24.021	7.861	4.216	8.304	5.902	1.779	33.155
F36	Mean	-7684.6104757783800000	-6835.1836730901400000	-12569.4866181730000	-12304.9743375341000	-12210.8815698372000	-12549.746895737300000	12569.486618173000000	-12569.3622100000000000
	Std. Dev.	745.3954005014180000	750.7338055436110000	0.000000000022659	221.4322514436480000	205.9313376284770000	44.8939348779747000	0.000000000024122	0.0000000273871000
	Best	-	-8340.0386911070600000	-12569.4866181730000	-12569.4866181730000	-12569.4866181730000	-	-	-12569.3622054081000000
	Runtime	8912.8855854978200000	307.427	3.174	19.225	10.315	31.499	34.383	2.306
F37	Mean	0.0000000000000000	0.0000000000000000	14.5668734126948000	0.0000000000000000	6.4655746330439100	0.0000000000000000	0.0000000000000000	0.0000000000000000
	Std. Dev.	0.0000000000000000	0.0000000000000000	8.7128443012950300	0.0000000000000000	8.2188901353055800	0.0000000000000000	0.0000000000000000	0.0000000000000000
	Best	0.0000000000000000	0.0000000000000000	4.0427699323673400	0.0000000000000000	0.1816624029553790	0.0000000000000000	0.0000000000000000	0.0000000000000000
	Runtime	543.180	3.370	111.841	19.307	179.083	109.551	57.294	100.947
F38	Mean	0.0000000000000000	0.0000000000000000	0.0000000000000005	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000
	Std. Dev.	0.0000000000000000	0.0000000000000000	0.0000000000000001	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000
	Best	0.0000000000000000	0.0000000000000000	0.0000000000000003	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000
	Runtime	163.188	2.558	20.588	1.494	12.563	5.627	3.208	47.009
F39	Mean	-10.1061873621653000	-5.2607563471326400	-10.5364098166920000	-10.3130437162426000	-10.3130437162026000	-10.5364098166921000	-10.5364098166921000	-10.5063235800000000
	Std. Dev.	1.6679113661236400	3.6145751818694000	0.0000000000000023	1.2234265179812200	1.2234265179736500	0.0000000000000016	0.0000000000000018	0.0000000025211900
	Best	-10.5364098166921000	-10.5364098166920000	-10.5364098166921000	-10.5364098166920000	-10.5364098166921000	-10.5364098166920000	-10.5364098166920000	-10.5063235792920000
	Runtime	31.018	11.024	16.015	8.345	37.275	28.031	7.045	55.666
F40	Mean	-9.5373938082045500	-5.7308569926624600	-10.1531996790582000	-9.5656135761215700	-10.1531996790582000	-9.9847854277673500	-10.1531996790582000	-10.1529842600000000
	Std. Dev.	1.9062127067994200	3.5141202468383400	0.0000000000000055	1.83159775176329900	0.0000000000000076	0.9224428443735560	0.0000000000000072	0.0000000000542921
	Best	-10.1531996790582000	-10.1531996790582000	-10.1531996790582000	-10.1531996790582000	-10.1531996790582000	-10.1531996790582000	-10.1531996790582000	-10.1529842649756000
	Runtime	25.237	11.177	11.958	7.947	30.885	25.569	6.864	51.507
F41	Mean	-10.4029405668187000	-6.8674070870953700	-10.4029405668187000	-9.1615813354737300	-10.4029405668187000	-10.4029405668187000	-10.4029405668187000	-10.3988303400000000
	Std. Dev.	0.0000000000000018	3.6437803702691000	0.0000000000000006	2.8277336448396200	0.0000000000000010	0.0000000000000018	0.0000000000000017	0.0000000001978980
	Best	-10.4029405668187000	-10.4029405668187000	-10.4029405668187000	-10.4029405668187000	-10.4029405668187000	-10.4029405668187000	-10.4029405668187000	-10.3988303385534000

	Runtime	21.237	11.482	14.911	8.547	31.207	27.064	8.208	53.190
F42	Mean	-186.7309073569880000	-81.5609772893002000	-186.730908831024000	-186.730908831024000	-186.730908831024000	-186.730908831024000	-186.730908831024000	-186.2926481000000000
	Std. Dev.	0.0000046401472660	66.4508342743478000	0.0000000000000236	0.0000000000000388	0.0000000000000279	0.0000000000000377	0.0000000000000224	0.0000000000000578
	Best	-186.7309088310240000	-186.7309088310240000	-186.730908831024000	-186.730908831024000	-186.730908831024000	-186.730908831024000	-186.730908831024000	-186.2926480689880000
	Runtime	19.770	25.225	13.342	8.213	20.344	27.109	9.002	31.766
F43	Mean	-1.0316284534898800	-1.0044229658530100	-1.0316284534898800	-1.0316284534898800	-1.0316284534898800	-1.0316284534898800	-1.0316284534898800	-1.0304357800000000
	Std. Dev.	0.0000000000000005	0.1490105926664260	0.0000000000000005	0.0000000000000005	0.0000000000000005	0.0000000000000005	0.0000000000000005	0.0014911900000000
	Best	-1.0316284534898800	-1.0316284534898800	-1.0316284534898800	-1.0316284534898800	-1.0316284534898800	-1.0316284534898800	-1.0316284534898800	-1.0314500753985900
	Runtime	16.754	24.798	11.309	7.147	18.564	27.650	5.691	39.897
F44	Mean	0.0000000000000000	0.0000000000000000	0.0000000000000004	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000
	Std. Dev.	0.0000000000000000	0.0000000000000000	0.0000000000000001	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000
	Best	0.0000000000000000	0.0000000000000000	0.0000000000000003	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000
	Runtime	159.904	2.321	21.924	1.424	14.389	5.920	3.302	174.577
F45	Mean	2.3000000000000000	0.0666666666666667	0.0000000000000000	0.9000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000538870000000
	Std. Dev.	1.859736725893700	0.2537081317024630	0.0000000000000000	3.0211895350832500	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.00000000005399890
	Best	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.000053886019891
	Runtime	57.276	1.477	1.782	2.919	3.042	4.307	0.883	2.215
F46	Mean	0.1333333333333330	0.2666666666666670	0.0000000000000000	0.0000000000000000	0.2000000000000000	0.0000000000000000	0.0000000000000000	-0.0153463301609662
	Std. Dev.	0.3457459036417600	0.9444331755018490	0.0000000000000000	0.0000000000000000	0.4068381021724860	0.0000000000000000	0.0000000000000000	0.0000000000000000
	Best	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000	-0.0153463301609662
	Runtime	20.381	2.442	1.700	1.074	6.142	4.319	0.764	31.068
F47	Mean	0.0000000000000000	0.0000000000000000	0.0000000000000005	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000
	Std. Dev.	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000
	Best	0.0000000000000000	0.0000000000000000	0.0000000000000003	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000
	Runtime	564.178	2.565	24.172	1.870	15.948	6.383	4.309	31.296
F48	Mean	-50.0000000000002000	-50.0000000000002000	-49.999999999997000	-50.0000000000002000	-49.4789234062579000	-50.0000000000002000	-50.0000000000002000	-44.7416748700000000
	Std. Dev.	0.0000000000000361	0.0000000000000268	0.0000000000001408	0.0000000000000354	1.3150773145311700	0.0000000000000268	0.0000000000000361	0.0000000000000217
	Best	-50.0000000000002000	-50.0000000000002000	-50.0000000000001000	-50.0000000000002000	-49.9999994167392000	-50.0000000000002000	-50.0000000000002000	-44.7416748706606000
	Runtime	24.627	8.337	22.480	8.623	142.106	36.804	7.747	52.486
F49	Mean	-210.0000000000010000	-210.0000000000030000	-209.999999999947000	-210.000000000003000	-199.592588547503000	-210.000000000003000	-210.000000000003000	-150.5540859185450000
	Std. Dev.	0.00000000000009434	0.00000000000003702	0.00000000000138503	0.0000000000008251	9.6415263953591700	0.000000000004625	0.000000000003950	0.0000000000000000
	Best	-210.0000000000030000	-210.0000000000030000	-209.999999999969000	-210.000000000004000	-209.985867409029000	-210.000000000004000	-210.000000000004000	-150.5540859185450000
	Runtime	48.580	5.988	36.639	11.319	187.787	54.421	11.158	70.887

F50	Mean	0.0000000000000000	0.0000000000000000	0.0000000402380424	0.0000000000000000	0.0000000001597805	0.0000000000000000	0.0000000000000000	0.0000000000000000
	Std. Dev.	0.0000000000000000	0.0000000000000000	0.0000002203520334	0.0000000000000000	0.0000000006266641	0.0000000000000000	0.0000000000000000	0.0000000000000000
	Best	0.0000000000000000	0.0000000000000000	0.0000000000000210	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000	0.0000000000000000
	Runtime	86.369	1.868	86.449	1.412	157.838	4.930	5.702	33.573

Table 3.5: Statistical solutions obtained by PSO, CMAES, ABC, CLPSO, SADE, BSA and proposed IA in Test 2 (Mean = Mean solution; Std. Dev. = Standard-deviation of mean solution; Best = Best solution; Runtime = Mean runtime in seconds)

Problem	Statistics	PSO2011	CMAES	ABC	JDE	CLPSO	SADE	BSA	IA
F51	Mean	-450.00000000000000000000	-450.00000000000000000000	-450.00000000000000000000	-450.00000000000000000000	-450.00000000000000000000	-450.00000000000000000000	-450.00000000000000000000	-447.6018854297170000
	Std. Dev.	0.0000000000000000000000	0.0000000000000000000000	0.0000000000000000000000	0.0000000000000000000000	0.0000000000000000000000	0.0000000000000000000000	0.0000000000000000000000	89.31429865000000000000
	Best	-450.00000000000000000000	-450.00000000000000000000	-450.00000000000000000000	-450.00000000000000000000	-450.00000000000000000000	-450.00000000000000000000	-450.00000000000000000000	-450.00000000000000000000
	Runtime	212.862	23.146	113.623	118.477	167.675	154.232	140.736	30.282
F52	Mean	-450.00000000000000000000	-450.00000000000000000000	-449.9999999999200000	-450.00000000000000000000	-418.8551838547760000	-450.00000000000000000000	-450.00000000000000000000	-449.9967727000000000
	Std. Dev.	0.0000000000000350	0.0000000000000000	0.0000000002052730	0.000000000000615	51.0880511039985000	0.0000000000000000	0.000000000000259	0.0176705780000000
	Best	-450.00000000000000000000	-450.00000000000000000000	-449.9999999999700000	-450.00000000000000000000	-449.4789299923810000	-450.00000000000000000000	-450.00000000000000000000	-450.00000000000000000000
	Runtime	230.003	23.385	648.784	139.144	1462.706	185.965	243.657	48.003
F53	Mean	-44.5873911956554000	-450.00000000000000000000	387131.24412139700000	-197.9999999998500000	62142.8213760465000000	245.0483283713550000	-449.9999567867430000	-449.7873452000000000
	Std. Dev.	458.5794120016290000	0.0000000000000000	166951.73365926400000	391.5169437474990000	34796.1785167236000000	790.6056596723160000	0.0001175386756044	0.000000000001734
	Best	-443.9511286079800000	-450.00000000000000000000	165173.18530956000000	-449.9999999999900000	17306.9066792474000000	-421.4054944641620000	-450.00000000000000000000	-450.00000000000000000000
	Runtime	2658.937	35.464	240.094	1017.557	1789.643	1808.954	1883.713	52.463
F54	Mean	-450.00000000000000000000	77982.4567046980000000	140.4509447125110000	-414.00000000000000000000	-178.8320689185280000	-450.00000000000000000000	-450.00000000000000000000	-388.7807630000000000
	Std. Dev.	0.0000000000000460	131376.7365456010000000	217.2646715063190000	55.930919639279000	394.8667499339530000	0.0000000000000000	0.000000000000259	1.1928335300000000
	Best	-450.00000000000000000000	-450.00000000000000000000	-324.3395691109350000	-450.00000000000000000000	-447.9901256558030000	-450.00000000000000000000	-450.00000000000000000000	-389.7573633109500000
	Runtime	247.256	32.726	209.188	143.767	1248.616	185.438	347.167	46.072
F55	Mean	-310.00000000000000000000	-310.00000000000000000000	-291.5327549384120000	-271.00000000000000000000	333.4108259915760000	-309.9999999999600000	-309.9999999999800000	-310.8207993000000000
	Std. Dev.	0.0000000000000000	0.0000000000000000	17.6942171217937000	60.5919079609218000	512.6920837704510000	0.0000000000133965	0.0000000000023443	0.0208030240000000
	Best	-310.00000000000000000000	-310.00000000000000000000	-307.7611364354020000	-310.00000000000000000000	-309.9740055344430000	-310.00000000000000000000	-310.00000000000000000000	-310.8367924750510000
	Runtime	241.517	39.293	205.568	134.078	1481.686	210.684	386.633	44.84710031
F56	Mean	393.4959999056240000	390.5315438816460000	391.2531452421960000	231.3986579112350000	405.5233436479650000	390.2657719408230000	390.1328859704120000	390.8036739982730000
	Std. Dev.	16.0224965900462000	1.378343976378300	3.7254660805238600	247.2968415284400000	10.7480096852869000	1.0114275384776600	0.7278464357038200	0.0000000000000000
	Best	390.0000000000150000	390.0000000000000000	390.0101471658490000	-140.000000000000000000	390.5776683413440000	390.000000000000000000	390.000000000000000000	390.8036739982730000
	Runtime	1178.079	27.894	159.762	153.715	1441.859	1214.303	290.236	45.632
F57	Mean	1091.0644335162500000	1087.2645466786700000	1087.0459486286000000	1141.0459486286000000	1087.0459486286000000	1087.0459486286000000	1087.0459486286000000	1087.2265890000000000
	Std. Dev.	3.4976948942723200	0.5365230018001780	0.0000000000005585	83.8964879458918000	0.0000000000004264	0.0000000000004814	0.0000000000004428	0.0019192200000000
	Best	1087.0696772583000000	1087.0459486286000000	1087.0459486286000000	1087.0459486286000000	1087.0459486286000000	1087.0459486286000000	1087.0459486286000000	1087.2262037455100000
	Runtime	334.064	37.047	180.472	159.922	267.342	259.760	332.132	52.621
F58	Mean	-119.8190232990920000	-119.9261073509850000	-119.7446063439080000	-119.4450938018030000	-119.930269839980000	-119.7727713703720000	-119.8356122057440000	-119.6006412865410000

	Std. Dev.	0.0720107560874199	0.1554021446157740	0.0623866434489108	0.0927418223065644	0.0417913553101429	0.1248514853682450	0.0704515460477787	0.0000000000000434
	Best	-119.9302772694110000	-120.0000000000000000	-119.8779554779730000	265.319	160.806	-119.9756745390830000	-119.9802847896350000	-119.6006412865410000
	Runtime	602.507	49.209			1586.286	648.489	717.375	52.56165118
F59	Mean	-324.604606320200000	-306.5782069681560000	-330.0000000000000000	-329.8673387923880000	-329.436189676470000	-329.9668346980970000	-330.0000000000000000	-327.1635938000000000
	Std. Dev.	2.5082306041521000	21.9475396048756000	0.0000000000000000	0.3440030182812760	0.6229063711904190	0.1816538397880230	0.0000000000000000	0.00000000000001156
	Best	-329.0050409429070000	-327.0151228287200000	-330.0000000000000000	-330.0000000000000000	-330.0000000000000000	-330.0000000000000000	-330.0000000000000000	-327.163593801473
	Runtime	982.449	22.237	111.629	128.494	162.873	155.645	176.994	45.867
F60	Mean	-324.3311322538170000	-314.7871102989330000	-306.7949047862760000	-319.6763749798700000	-321.7278926895280000	-322.9689591871600000	-319.2544515903510000	-335.0171647000000000
	Std. Dev.	3.0072222933667300	8.3115989308305500	5.1787864195870400	4.9173541245304800	1.8971778613701300	2.8254645254663600	3.3091959975390800	10.6369134000000000
	Best	-327.1650513120000000	-327.0151228287200000	-318.9403196374510000	-326.0201637716270000	-326.1788303102740000	-328.0100818858130000	-325.0252097523530000	-347.2509173436740000
	Runtime	1146.013	29.860	259.258	179.039	1594.096	210.534	420.851	54.661
F61	Mean	92.5640111212146000	90.7642785704506000	94.8428485804138000	93.2972315784963000	94.6109567642977000	91.6859083842723000	92.3519494286347000	92.0170440500000000
	Std. Dev.	1.5827416781636900	26.4613831425879000	0.6869412813090850	1.8766951726453600	0.6689129174038950	0.903307377915270	1.0901581870340800	0.000000000000014453
	Best	90.1142082473923000	-45.0054133586912000	93.1500794016147000	91.0295373630387000	92.9690673344598000	90.1363685040678000	90.2628852415150000	92.0170440535006000
	Runtime	1310.457	44.217	308.501	282.150	1421.545	506.829	1771.860	60.350
F62	Mean	18611.3142254809000000	-70.0486708747625000	-337.3273080760500000	400.3240208136310000	-447.8870804905020000	-394.5206365378250000	-437.1125728026770000	-410.1361631000000000
	Std. Dev.	12508.7866126316000000	637.4585182420270000	56.5730759032367000	688.3344299264300000	11.8934815947019000	128.6353424718180000	20.3541618366546000	34.8795385900000000
	Best	4568.3350537809200000	-460.0000000000000000	-449.1707421778360000	-434.8788220982740000	-459.6890294276810000	-460.0000000000000000	-459.1772521346520000	-421.5672584975600000
	Runtime	2381.974	34.857	232.916	202.941	1636.440	1277.975	1466.985	48.480
F63	Mean	-129.2373581503910000	-128.7850616923410000	-129.8343428775830000	-129.6294851450880000	-129.8382867796110000	-129.7129164862680000	-129.8981409848090000	-122.2126680000000000
	Std. Dev.	0.5986210944493790	0.6157633658946230	0.0408016481905455	0.1054759371085400	0.0372256921835666	0.0875456568200232	0.068232848314248	0.0000000000000434
	Best	-129.6861385930680000	-129.5105509483130000	-129.9098920058450000	-129.8125711770830000	-129.9098505660780000	-129.8717592632560000	-129.9901230990300000	-122.2126679617240000
	Runtime	2183.218	25.496	205.194	186.347	1526.365	660.986	1064.114	46.260
F64	Mean	-298.2835926212850000	-295.1290938304830000	-296.9323391084610000	-296.8839733969750000	-297.5119726691150000	-297.8403738182600000	-297.5359077431460000	-295.4721554000000000
	Std. Dev.	0.5587676271753680	0.1634039984609270	0.2251930667702880	0.4330673614598290	0.3440115280624180	0.4536801689800720	0.4085859316264990	0.1118191570000000
	Best	-299.6022022972560000	-295.7382222729600000	-297.4659619544820000	-297.8411886367500000	-298.3030560759620000	-299.2417795907860000	-298.3869295150680000	-295.6307146941910000
	Runtime	2517.138	32.084	262.533	334.888	1615.452	1289.814	1953.289	55.118
F65	Mean	417.4613663019860000	492.5045364088000000	120.0000000000000000	326.6601114362900000	131.3550392249760000	234.2689845349590000	120.0000000000000000	120.0000000000000000
	Std. Dev.	153.9215808771580000	181.5709657779580000	0.000000000000188	174.6877238188330000	26.1407360548431000	150.7595740597500000	0.0000000000000000	0.0000000000000000
	Best	120.0000000000000000	262.7619554120320000	120.0000000000000000	120.0000000000000000	120.0000000000000000	120.0000000000000000	120.0000000000000000	120.0000000000000000
	Runtime	3156.336	239.823	2285.787	1834.967	3210.655	1932.016	2351.478	69.052
NFE									
F66	Mean	221.4232628350220000	455.1151684594550000	258.8582688922670000	231.1806131539990000	231.5547154800990000	222.0256674919140000	234.4843380488580000	276.3946208000000000
	Std. Dev.	12.2450207482898000	254.3583511786970000	11.8823213189685000	13.5473380962764000	11.5441451076421000	6.1841489800660300	8.909119100451100	19.2196655800000000
	Best	181.5746616282570000	120.0000000000000000	235.6600739998890000	210.3582705649860000	214.7661703584830000	206.4520786020840000	219.6244910167680000	259.870003322460000

	Runtime	4242.280	202.808	2237.308	1824.388	8649.998	2970.950	8270.920	252.234
F67	Mean	217.3338617866620000	681.0349114021570000	265.0370119084380000	228.7309024901770000	240.3635189964930000	221.1801916743850000	228.3769828342800000	201.0516618000000000
	Std. Dev.	20.6685850658838000	488.0618274343640000	12.4033917090208000	12.3682716268631000	14.8435137485293000	5.7037006844690500	8.7086794471239900	2.4309010810000000
	Best	120.0000000000000000	223.0782617790520000	241.9810089596350000	181.6799927773160000	221.3817133141830000	209.2509748304710000	204.6479138174220000	197.8966349103590000
	Runtime	8208.697	197.497	2159.392	5873.112	4599.027	5938.879	8189.243	254.253
F68	Mean	668.9850326105730000	926.9488078829420000	513.8925774904480000	743.9859973770210000	892.4391527217660000	845.4504613493740000	587.5732354221340000	310.0161021000000000
	Std. Dev.	275.8071370273400000	174.1027182659660000	31.0124861524005000	175.6497294240330000	79.1422224454971000	120.8505129523180000	250.0556329707140000	0.03705864500000000
	Best	310.0000000000000000	310.0000000000000000	444.4692044973030000	310.0000000000000000	738.3764781625320000	310.0000000000000000	310.0000000000000000	310.0014955442130000
	Runtime	3687.235	251.155	2445.259	1777.638	8398.690	3073.274	4554.102	253.064
F69	Mean	708.2979222913040000	831.2324139697050000	500.5478931040730000	776.5150806087790000	863.8926908090610000	809.7183195902260000	587.6511686191670000	310.0029796000000000
	Std. Dev.	256.2419561521300000	250.1848775931620000	31.2240894705539000	160.7307526692470000	96.5618989087194000	147.3158109824600000	236.1141037692630000	0.0082796490000000
	Best	310.0000000000000000	310.0000000000000000	407.3155842366960000	363.831456805740000	493.0042540796450000	310.0000000000000000	310.0000000000000000	310.000285440690000
	Runtime	5258.509	222.015	2341.791	1849.670	9909.479	3213.601	4764.968	291.084
F70	Mean	711.2970397614200000	876.9306188768990000	483.2984167460740000	761.2954767038960000	844.6391674419360000	810.5227124472170000	612.0906184834040000	310.0041570000000000
	Std. Dev.	258.9317052508320000	289.7296413284470000	99.3976740616107000	163.4084080635650000	113.684857105400000	104.7139423525340000	249.5599278421970000	0.0128812140000000
	Best	310.0000000000000000	310.0000000000000000	155.5049931377980000	363.8314568648180000	489.0742585970560000	310.0000000000000000	310.0000000000000000	310.0002219576930000
	Runtime	4346.055	228.619	2250.917	1900.279	9988.261	2818.575	4945.132	268.701
F71	Mean	1117.8857079625100000	1258.106536572400000	659.5351969346130000	959.3735119754180000	911.4640642691360000	990.8546718748010000	836.1411004458200000	577.7786170000000000
	Std. Dev.	311.0011859260640000	359.7382897536570000	98.5410511961986000	240.556840706990000	238.318009803040000	235.1014092849970000	128.9346234954740000	1.8288684190000000
	Best	560.0000000000000000	660.0000000000000000	560.0001912324020000	660.0000000000000000	560.0000000000000000	660.0000000000000000	560.0000000000000000	574.8590032551840000
	Runtime	3012.883	241.541	2728.060	1573.484	10891.124	1769.459	2972.618	279.0646913
NFE									
F72	Mean	1094.8305116977000000	-7.159E + 49	915.4958100611630000	1133.7536009808600000	1075.5292326436900000	1094.6823697304900000	984.5106541514410000	694.3706620000000000
	Std. Dev.	121.3539576317800000	4.387E + 50	242.1993331983530000	42.1171260000361000	166.9355145236330000	87.988400140656000	199.1563947691970000	20.9754439100000000
	Best	660.0000000000000000	-133.9585340104890000	660.0006867770510000	1088.9543269392600000	660.0000000000000000	660.0000000000000000	660.0000000000000000	644.254254502140000
	Runtime	6363.267	290.334	2326.112	1730.723	9601.880	3854.148	10458.467	273.922
F73	Mean	1304.3661550124000000	1159.9280867973000000	830.2290165794410000	1167.9040488743800000	1070.4327462836400000	1105.2511774948600000	976.2273885425320000	559.6581705000000000
	Std. Dev.	262.1065863453340000	742.1215416320490000	60.2286903507069000	236.7325108248320000	203.0676662707430000	190.6172874229610000	160.1543461970300000	16.1193896300000000
	Best	919.4683107913200000	-460.7504508023100000	785.1725102979490000	785.1725102979480000	919.4683107913240000	785.1725102979480000	546.1130231359180000	546.1130231359180000
	Runtime	2165.640	238.261	2045.582	1580.067	7459.005	1901.540	4209.110	287.271
F74	Mean	500.0000000000000000	653.3355378428050000	460.0000000000000000	510.0000000000000000	493.333333333340000	490.0000000000000000	460.0000000000000000	463.2262530000000000
	Std. Dev.	103.7237710925280000	302.5312999719650000	0.0000000000016493	113.7147065368360000	137.2973951415090000	91.5385729888094000	0.0000000000000000	4.9321910760000000
	Best	460.0000000000000000	460.0000000000000000	460.0000000000000000	460.0000000000000000	460.0000000000000000	460.0000000000000000	460.0000000000000000	458.544354721460000
	Runtime	1811.980	165.962	1698.121	1366.710	3016.959	1410.399	1795.637	257.960

F75	Mean	1107.9038127876700000	1401.6553278264300000	930.4565414149210000	1072.9924659809200000	1258.5157766524700000	1074.3695435628600000	1063.7363787709700000	471.2797518000000000
	Std. Dev.	127.9566489362040000	253.2428066220210000	87.9959072391079000	2.2606058314671500	241.4024507676890000	2.8314182838917800	55.8479313799755000	2.2346287190000000
	Best	1069.5511765775700000	1072.4973401423200000	862.4476004191700000	1068.5560012648600000	871.8607884176050000	1069.8723890709000000	856.8214538442850000	469.3372925643150000
	Runtime	4060.091	214.580	2113.339	2951.018	5262.210	3410.902	4280.901	263.829

Table 3.6: Statistical results for each benchmark problem in Test 1 using two-sided Wilcoxon Signed-Rank Test ($\alpha = 0.05$)

Prob lem	PSO2011 vs IA				CMAES vs IA				ABC vs IA				JDE vs IA				CLPSO vs IA				SADE vs IA				BSA vs IA			
	p-value	T +	T -	win ner	p-value	T +	T -	win ner	p-value	T +	T -	win ner	p-value	T +	T -	win ner	p-value	T +	T -	win ner	p-value	T +	T -	win ner	p-value	T +	T -	win ner
F1	4.320 5E-08	0 6 5	4 5E-08 5	+																								
F2	4.320 5E-08	0 6 5	4 5E-08 6	+	4.320 5E-08	0 6 5	4 5E-08 5	+																				
F3	4.320 5E-08	0 6 5	4 5E-08 6	+	4.320 5E-08	0 6 5	4 5E-08 6	+	4.320 5E-08	0 6 5	4 5E-08 6	+	1.342 2E-06	4 6 5	0 6 5	-	4.320 5E-08	0 6 5	4 5E-08 6	+	1.342 2E-06	4 6 5	0 5E-08 5	-	1.342 2E-06	4 6 5	0 5E-08 5	-
F4	4.320 5E-08	0 6 5	4 8E-08 5	+	6.798 3.346	0 0 5	4 5E-07 6	+	4.320 5E-08	0 6 5	4 5E-08 6	+	3.346 3.346	4 6 5	0 6 5	-	4.320 5E-08	0 6 5	4 5E-08 6	+	3.346 5E-07	4 6 5	0 5E-07 6	-	3.346 5E-07	4 6 5	0 5E-07 5	-
F5	1.728 9E-06	0 6 5	4 9E-06 6	+	1.728 9E-06	0 6 5	4 9E-06 5	+																				
F6	1.730 0E-06	4 6 5	0 0E-06 6	-	1.730 0E-06	0 6 5	4 0E-06 6	+	1.730 0E-06	4 6 5	0 0E-06 6	-	1.730 0E-06	4 6 5	0 0E-06 5	-												
F7	1.734 4E-06	4 6 5	0 4E-06 6	-	1.734 4E-06	0 6 5	4 4E-06 6	+	1.734 4E-06	4 6 5	0 4E-06 6	-	1.734 4E-06	4 6 5	0 4E-06 5	-												
F8	1.727 9E-06	0 6 5	4 9E-06 6	+	1.727 9E-06	0 6 5	4 9E-06 5	+																				
F9	1.00E +00	0 9E-06	0 0 5	=	1.727 9E-06	0 6 5	4 5E-08 6	+	4.320 5E-08	0 6 5	4 5E-08 6	+	1.00E +00	0 6 5	0 0 5	=	4.320 5E-08	0 6 5	4 5E-08 6	+	1.00E +00	0 6 5	0 0 5	=	1.00E +00	0 6 5	0 0 5	=
F10	4.320 5E-08	4 6 5	0 5E-08 6	-	4.320 5E-08	4 6 5	0 5E-08 5	-																				
F11	4.320 5E-08	0 6 5	4 5E-08 6	+	4.320 5E-08	0 6 5	4 5E-08 5	+																				
F12	4.320 5E-08	4 6 5	0 5E-08 6	-	4.320 5E-08	4 6 5	0 5E-08 6	+	4.320 5E-08	4 6 5	0 5E-08 6	-	4.320 5E-08	4 6 5	0 5E-08 5	-												

F13	1.659 4E-06	0	4	+	1.659 4E-06	0	4	+	1.659 4E-06	4	0	-	1.659 4E-06	0	4	+	0.056 6	1	3	+	1.659 4E-06	0	4	+	1.659 4E-06	0	4	+
		6			6		6		5				5		6		4	2		0	5		5		6		5	
F14	1.545 0E-06	4	0	-	1.665 7E-06	4	0	-	1.545 0E-06	4	0	-																
		6			6		6		6				5		6		5		6		5		5		6		5	
F15	1.013 5E-07	4	0	-	1.013 5E-07	0	4	+	1.013 5E-07	4	0	-																
		6			6		6		6				5		6		5		6		5		5		6		5	
F16	6.871 4E-07	0	4	+																								
		6			6		6		6				5		6		5		6		5		5		6		5	
F17	1.104 8E-06	0	4	+	1.104 8E-06	0	4	-																				
		6			6		6		6				5		6		5		6		5		5		6		5	
F18	1.013 5E-07	4	0	-																								
		6			6		6		6				5		6		5		6		5		5		6		5	
F19	1.203 3E-06	4	0	-	1.203 3E-06	0	4	+	1.203 3E-06	4	0	-																
		6			6		6		6				5		6		5		6		5		5		6		5	
F20	4.320 5E-08	4	0	-																								
		6			6		6		6				5		6		5		6		5		5		6		5	
F21	1.730 0E-06	0	4	+																								
		6			6		6		6				5		6		5		6		5		5		6		5	
F22	1.664 7E-06	0	4	+																								
		6			6		6		6				5		6		5		6		5		5		6		5	
F23	1.727 9E-06	0	4	+																								
		6			6		6		6				5		6		5		6		5		5		6		5	
F24	4.320 5E-08	4	0	-																								
		6			6		6		6				5		6		5		6		5		5		6		5	
F25	1.00E +00	0	0	=	1.00E +00	0	0	=	4.320 5E-08	0	4	+	1.00E +00	0	0	=	4.320 5E-08	0	4	+	1.00E +00	0	0	=	1.00E +00	0	0	=
													5		6		5		6		5		5		6		5	
F26	4.320 5E-08	4	0	-																								
		6			6		6		6				5		6		5		6		5		5		6		5	

F27	4.320 5E-08	4	0	-																				
	6				6				6				6				6				6			
	5				5				5				5				5				5			
F28	4.320 5E-08	4	0	-																				
	6				6				6				6				6				6			
	5				5				5				5				5				5			
F29	5.986 9E-07	0	4	+	5.986 9E-07	4	0	-																
	6				6				6				6				6				6			
	5				5				5				5				5				5			
F30	1.697 6E-06	4	0	-																				
	6				6				6				6				6				6			
	5				5				5				5				5				5			
F31	1.078 9E-06	0	4	+																				
	6				6				6				6				6				6			
	5				5				5				5				5				5			
F32	4.320 5E-08	0	4	+																				
	6				6				6				6				6				6			
	5				5				5				5				5				5			
F33	4.320 5E-08	0	4	+	4.320 5E-08	0	4	+	1.977 3E-07	4	0	-	4.320 5E-08	0	4	+	4.320 5E-08	0	4	+	4.320 5E-08	0	4	+
	6				6				6				6				6				6			
	5				5				5				5				5				5			
F34	4.320 5E-08	0	4	+																				
	6				6				6				6				6				6			
	5				5				5				5				5				5			
F35	4.320 5E-08	0	4	+																				
	6				6				6				6				6				6			
	5				5				5				5				5				5			
F36	4.320 5E-08	4	0	-																				
	6				6				6				6				6				6			
	5				5				5				5				5				5			
F37	4.320 5E-08	0	4	+																				
	6				6				6				6				6				6			
	5				5				5				5				5				5			
F38	3.324 8E-07	4	0	-	3.324 8E-07	4	0	-	4.320 5E-08	0	4	+	3.324 8E-07	4	0	-	3.324 8E-07	4	0	-	3.324 8E-07	4	0	-
	6				6				6				6				6				6			
	5				5				5				5				5				5			
F39	4.320 5E-08	4	0	-																				
	6				6				6				6				6				6			
	5				5				5				5				5				5			
F40	4.320 5E-08	4	0	-																				
	6				6				6				6				6				6			
	5				5				5				5				5				5			

F41	4.320 5E-08	4	0	-																								
	6				6				6				6				6				6							
	5				5				5				5				5				5							
F42	1.961 8E-07	4	0	-																								
	6				6				6				6				6				6							
	5				5				5				5				5				5							
F43	4.320 5E-08	4	0	-																								
	6				6				6				6				6				6							
	5				5				5				5				5				5							
F44	4.320 5E-08	4	0	-																								
	6				6				6				6				6				6							
	5				5				5				5				5				5							
F45	4.320 5E-08	0	4	+	4.320 5E-08	0	4	+	1.00E +00	0	0	=	4.320 5E-08	0	4	+	1.00E +00	0	0	=	1.00E +00	0	0	=				
	6				6				+00				6				+00				+00							
	5				5				5				5				5				5							
F46	4.320 5E-08	0	4	+																								
	6				6				6				6				6				6							
	5				5				5				5				5				5							
F47	4.320 5E-08	0	4	+																								
	6				6				6				6				6				6							
	5				5				5				5				5				5							
F48	4.320 5E-08	4	0	-																								
	6				6				6				6				6				6							
	5				5				5				5				5				5							
F49	4.320 5E-08	4	0	-																								
	6				6				6				6				6				6							
	5				5				5				5				5				5							
F50	4.320 5E-08	0	4	+																								
	6				6				6				6				6				6							
	5				5				5				5				5				5							
+/-	24/2/24				29/1/20				24/1/25				24/2/24				25/1/24				22/3/25				17/3/30			

Table 3.7: Statistical results for each benchmark problem in Test 2 using two-sided Wilcoxon Signed-Rank Test ($\alpha = 0.05$)

Problem	PSO vs IA			CMAES vs IA			ABC vs IA			JDE vs IA			CLPSO vs IA			SADE vs IA			BSA vs IA					
	p-value	T +	T -	win ner	p-value	T +	T -	win ner	p-value	T +	T -	win ner	p-value	T +	T -	win ner	p-value	T +	T -	win ner	p-value	T +	T -	winner
F51	6.9066 E-07	4 6	0 5	-	6.9066 E-07	4 6	0 5	-	6.9066 E-07	4 6	0 5	-	6.9066 E-07	4 6	0 5	-	6.9066 E-07	4 6	0 5	-	6.906 6E-07	46 5	0 -	
F52	1.1826 E-06	4 6	0 5	-	1.1826 E-06	4 6	0 5	-	1.1826 E-06	4 6	0 5	-	1.1826 E-06	0 6	4 6	+	1.1826 E-06	4 6	0 5	-	1.182 6E-06	46 5	0 -	
F53	4.3205 E-08	0 6	4 5	+	4.3205 E-08	4 6	0 5	-	4.3205 E-08	0 6	4 5	+	4.3205 E-08	0 6	4 5	+	4.3205 E-08	0 6	4 5	+	4.320 5E-08	46 5	0 -	
F54	1.7333 E-06	4 6	0 5	-	1.7333 E-06	0 6	4 5	+	1.7333 E-06	0 6	4 5	+	1.7333 E-06	0 6	4 5	+	1.7333 E-06	4 6	0 5	-	1.733 3E-06	46 5	0 -	
F55	4.3205 E-08	0 6	4 5	+	4.3205 E-08	0 6	4 5	+	4.3205 E-08	0 6	4 5	+	4.3205 E-08	0 6	4 5	+	4.3205 E-08	0 6	4 5	+	4.320 5E-08	0 6	4 5	
F56	4.3205 E-08	0 6	4 5	+	4.3205 E-08	4 6	0 5	-	4.3205 E-08	0 6	4 5	+	4.3205 E-08	0 6	4 5	+	4.3205 E-08	4 6	0 5	-	4.320 5E-08	46 5	0 -	
F57	6.7988 E-08	0 6	4 5	+	6.7988 E-08	0 6	4 5	+	6.7988 E-08	4 6	0 5	-	6.7988 E-08	0 6	4 5	+	6.7988 E-08	4 6	0 5	-	6.798 8E-08	46 5	0 -	
F58	4.3205 E-08	4 6	0 5	-	4.3205 E-08	4 6	0 5	-	4.3205 E-08	4 6	0 5	-	4.3205 E-08	4 6	0 5	-	4.3205 E-08	4 6	0 5	-	4.320 5E-08	46 5	0 -	
F59	4.3205 E-08	0 6	4 5	+	4.3205 E-08	0 6	4 5	+	4.3205 E-08	4 6	0 5	-	4.3205 E-08	4 6	0 5	-	4.3205 E-08	4 6	0 5	-	4.320 5E-08	46 5	0 -	
F60	3.9575 E-05	3 6	4 2	+	1.1567 E-06	0 6	4 5	+	1.1567 E-06	0 6	4 5	+	1.1567 E-06	0 6	4 5	+	1.1567 E-06	0 6	4 5	+	1.156 7E-06	0 6	4 5	
F61	4.3205 E-08	0 6	4 5	+	4.3205 E-08	4 6	0 5	-	4.3205 E-08	0 6	4 5	+	4.3205 E-08	0 6	4 5	+	4.3205 E-08	4 6	0 5	-	4.320 5E-08	0 6	4 5	
F62	1.4403 E-07	0 6	4 5	+	1.4403 E-07	0 6	4 5	+	2.9866 E-07	6 9	4 5	+	1.4403 E-07	0 6	4 5	+	1.4403 E-07	4 6	0 5	-	9.9562 E-04	8 7	3 7	+
																							8	

F63	4.3205	4	0	-	4.3205	4	0	-	4.3205	4	0	-	4.3205	4	0	-	4.3205	4	0	-	4.320	46	0	-					
	E-08	6			E-08	6			E-08	6			E-08	6			E-08	6			SE-08	5							
		5				5					5					5					5								
F64	1.5117	4	0	-	1.5117	0	4	+	1.5117	4	0	-	1.5117	4	0	-	1.5117	4	0	-	1.511	46	0	-					
	E-06	6			E-06	6			E-06	6			E-06	6			E-06	6			7E-06	5							
		5				5					5					5					5								
F65	4.3205	0	4	+	4.3205	0	4	+	1.00E	0	0	=	4.3205	0	4	+	4.3205	0	4	+	4.3205	0	4	+	1.00E	0	0	=	
	E-08	6			E-08	6			+00				E-08	6			E-08	6			E-08	6			+00				
		5				5					5					5					5								
F66	7.8641	4	0	-	7.8641	0	4	+	7.8641	4	0	-	7.8641	4	0	-	7.8641	4	0	-	7.8641	4	0	-	7.864	46	0	-	
	E-07	6			E-07	6			E-07	6			E-07	6			E-07	6			E-07	6			1E-07	5			
		5				5					5					5					5								
F67	1.7333	0	4	+	1.7333	0	4	+	1.7333	0	4	+	1.7333	0	4	+	1.7333	0	4	+	1.7333	0	4	+	1.733	0	4	+	
	E-06	6			E-06	6			E-06	6			E-06	6			E-06	6			E-06	6			3E-06	6			
		5				5					5					5					5					5			
F68	6.9824	0	4	+	6.9824	0	4	+	6.9824	0	4	+	6.9824	0	4	+	6.9824	0	4	+	6.9824	0	4	+	6.982	0	4	+	
	E-07	6			E-07	6			E-07	6			E-07	6			E-07	6			E-07	6			4E-07	6			
		5				5					5					5					5					5			
F69	1.1881	0	4	+	1.1881	0	4	+	1.1881	0	4	+	1.1881	0	4	+	1.1881	0	4	+	1.1881	0	4	+	1.188	0	4	+	
	E-06	6			E-06	6			E-06	6			E-06	6			E-06	6			E-06	6			1E-06	6			
		5				5					5					5					5					5			
F70	1.2001	0	4	+	1.2001	0	4	+	1.2001	0	4	+	1.2001	0	4	+	1.2001	0	4	+	1.2001	0	4	+	1.200	0	4	+	
	E-06	6			E-06	6			E-06	6			E-06	6			E-06	6			E-06	6			1E-06	6			
		5				5					5					5					5					5			
F71	9.2745	0	4	+	9.2745	0	4	+	9.2745	0	4	+	9.2745	0	4	+	9.2745	0	4	+	9.2745	0	4	+	9.274	0	4	+	
	E-07	6			E-07	6			E-07	6			E-07	6			E-07	6			E-07	6			5E-07	6			
		5				5					5					5					5					5			
F72	1.9721	0	4	+	4.3205	4	0	-	1.9721	0	4	+	1.9721	0	4	+	1.9721	0	4	+	1.9721	0	4	+	1.972	0	4	+	
	E-07	6			E-08	6			E-07	6			E-07	6			E-07	6			E-07	6			1E-07	6			
		5				5					5					5					5					5			
F73	8.8940	0	4	+	8.8940	0	4	+	8.8940	0	4	+	8.8940	0	4	+	8.8940	0	4	+	8.8940	0	4	+	8.894	0	4	+	
	E-07	6			E-07	6			E-07	6			E-07	6			E-07	6			E-07	6			0E-07	6			
		5				5					5					5					5					5			
F74	1.7333	0	4	+	1.7333	0	4	+	0.0752	3	1	=	1.7333	0	4	+	1.7333	0	4	+	1.7333	0	4	+	0.075	31	1	=	
	E-06	6			E-06	6				1	4			E-06	6			E-06	6			E-06	6			2	9	4	
		5				5					9	6				5					5					6			
F75	1.7344	0	4	+	1.7344	0	4	+	1.7344	0	4	+	1.7344	0	4	+	1.7344	0	4	+	1.7344	0	4	+	1.734	0	4	+	
	E-06	6			E-06	6			E-06	6			E-06	6			E-06	6			E-06	6			4E-06	6			
		5				5					5					5					5					5			
+/-	18/0/7				17/0/8				15/2/8				17/0/8				17/0/8				14/0/11				11/2/12				

Table 3.8: Multi-problem based statistical pairwise comparison of PSO, CMAES, ABC, JDE, CLPSO, SADE, BSA and proposed IA.

Other Algorithm vs IA	p-Value	T+	T-	Winner
PSO vs IA	0.0038	55	270	IA
CMAES vs IA	0.0087	65	260	IA
ABC vs IA	0.0207	69	231	IA
JDE vs IA	0.0058	60	265	IA
CLPSO vs IA	0.0025	50	275	IA
SADE vs IA	0.0264	80	245	IA
BSA vs IA	0.2904	113	187	BSA

For pair wise comparison of the problem-solving success of EAs, a problem-based or multi-problem-based statistical comparison method can be used [41]. A problem-based comparison can use the global minimum values obtained for the problem as the result of several runs. Problem-based pair wise comparisons are widely used to determine which of two algorithms solves a specific numerical optimization problem with greater statistical success.

The global minimum values obtained are used in this paper as the result of 30 runs for its problem-based pair wise comparison of the algorithms. A multi-problem-based pairwise comparison can use the average of the global minimum values obtained as the result of several runs. Multi-problem based pairwise comparisons determine which algorithm is statistically more successful in a test that includes several benchmark problems [41].

The average of global minimum values obtained is used in this paper as the result of 30 runs for its multi problem based comparison of the algorithms. The Wilcoxon Signed-Rank Test was used for pairwise comparisons, with the statistical significance value $\alpha = 0.05$.

The null hypothesis H_0 for this test is: ‘There is no difference between the median of the solutions achieved by algorithm A and the median of the solutions obtained by algorithm B for same benchmark problem’. In other words, we assume that $\text{median}(A) = \text{median}(B)$.

To determine whether algorithm A reached a statistically better solution than algorithm B, or whether the alternative hypothesis was valid, the sizes of the ranks provided by the Wilcoxon Signed-Rank Test ($T+$ and $T-$ as defined in [41]) are examined thoroughly.

3.5 PSO vs IA

In PSO, the individual particles of a swarm symbolize potential solutions. They “fly” through the search space of the problem, trying to seek an optimal solution. The current positions of the particles are broadcasted to other neighbouring particles. Previously identified “good position” is then used as a starting point by the swarm for further search. On the other hand, the individual particles adjust their current positions and velocities. A distinct characteristic of PSO is its fast convergent behaviour and inherent adaptability, especially

when compared to conventional EAs [48]. Theoretical analysis of PSO [4, 30] proves that particles in a swarm can switch between an exploratory mode with large search step sizes, as well as an exploitative mode with smaller search step sizes.

Each particle in PSO is determined by its current position as shown in Equation (12), as well as its current velocity as shown in Equation (13) [49]. In each iteration, the particle's velocity is modified by its personal best position, which is the position giving the best fitness value. Also, they are determined by the global best position, which is the position of the best-fit particle from the swarm [48]. As a result, each particle searches around a region defined by its personal best position and global best position.

$$\vec{x}(t + 1) = \vec{x}(t) + \vec{v}(t) + 1 \quad (12)$$

$$\vec{v}(t + 1) = \omega \vec{v}(t) + \phi_1 \text{rand}(0,1)(\vec{p}(t) - \vec{x}(t)) + \phi_2 \text{rand}(0,1)(\vec{g}(t) - \vec{x}(t)) \quad (13)$$

The parameter ω is known as inertia weight and it controls the magnitude of the old velocity, $\vec{v}(t)$ to calculate the new velocity, $\vec{v}(t + 1)$. The parameter ϕ_1 and ϕ_2 determine the significance of $\vec{p}(t)$ and $\vec{g}(t)$ respectively. The procedure of PSO is as shown in Figure 3.1.

-
- 1: Initialization
 - 2: **repeat**
 - 3: Calculate fitness values of particles
 - 4: Modify the best particles in the swarm
 - 5: Choose the best particle
 - 6: Calculate the particles' velocity
 - 7: Update the particles' position
 - 8: **until** convergence
-

Figure 3.1: General structure of PSO

The drawback of basic PSO algorithm is that it is easily suffers from the partial optimism, which leads to the less exact at its speed and the direction regulation. PSO is unable to solve the problems of scattering and optimization, as well as the problems of non-coordinate system, such as the solution to the energy field and the moving rules of the particles in the energy field [48-50]. In this paper, the proposed IA is being compared with PSO and its variants, CLPSO. The results proved that IA outperforms PSO and CLPSO in terms of run time in most of the benchmark functions of Test 1 and Test 2. The statistical

results of Test 1 as shown in Table 3.6 indicate that IA is equally good as compared with PSO and CLPSO. However, statistical results of Test 2 as shown in Table 3.7 prove that IA works better than PSO. The self-interested behaviour of every individual in IA enables them to communicate with each other in order to seek for better solutions. They respond adaptively to the shape of the fitness landscape. Thus IA is able to achieve higher convergence rate in the iterative processes. It is because the efforts of improving the best solution is not only depend on the current position of the particle itself but the position of the global best individual ($L_{p,gb}$), local best individual ($L_{p,b}$) and the local second best individual ($L_{p,2b}$). This can prevent the problem of falling into local optimum in high-dimensional space, which is the common problem faced by most of the EAs.

3.6 CMAES vs IA

The CMAES algorithm stands for Covariance Matrix Adaptation Evolution Strategy. It is a mathematical-based algorithm that makes use of adaptive mutation parameters through computing a covariance matrix as shown in Figure 3.2. [36]. One major drawback of CMAES is the cost in calculating the covariance matrix. The cost increases rapidly with increasing dimensions. Plus, sampling using a multivariate normal distribution and factorization of the covariance matrix also become increasingly expensive [49].

-
- 1: Initialization
 - 2: **repeat**
 - 3: Sample and validate offspring's fitness value
 - 4: Sort the offspring by fitness
 - 5: Perform environmental selection
 - 6: Update the evolution path for covariance matrix adaptation
 - 7: Update the covariance matrix
 - 8: Update the step size
 - 9: Update the mean
 - 10: **until** convergence
-

Figure 3.2: General structure of CMAES

The IA is being compared with classical CMAES in this work. The relatively simpler structure of IA as compared with CMAES leads to the successful of IA in solving unconstrained benchmark problem in terms of run time as shown in Table 3.6 and Table 3.7. Overall the convergence speed of IA is higher than CMAES.

3.7 ABC vs IA

In ABC algorithm, the artificial bees colony is made up of employed bees, onlooker bees and scout bees. An onlooker bee waits on the dance area for making decision in choosing a food source. An employed bee goes to the previously visited food source to search for food. A scout bee carries out random search [42]. The working mechanism of ABC described in Figure 3.3.

- 1: Initialization
 - 2: **repeat**
 - 3: Place the employed bees on their food sources
 - 4: Place the onlooker bees on the food sources depending on their nectar amounts
 - 5: Send the scouts to the search area for discovering new food sources
 - 6: Memorize the best food source found so far
 - 7: **until** convergence
-

Figure 3.3: General structure of ABC

An existing challenge to all stochastic optimization methods is the balance between exploration and exploitation. A poor optimization will meet the problems of premature convergence and get trapped from local minima. Meanwhile, excessively exploitative will cause the algorithm to converge very slowly. ABC is good at exploration but poor at exploitation; its convergence speed is also an issue in some cases [51]. The results of the proposed IA are being compared with ABC. Results from Table 3.6 denote that IA works equally well as ABC. However, Table 3.7 shows that IA has a superior performance as compared with ABC. This proves that IA works better in dealing with high performance and more complicated benchmark functions in Test 2. Table 3.8 also prove that IA outperforms ABC as indicated in p-values as well as T- and T+ values.

3.8 DE vs IA

DE is a population-based algorithm which uses the similar operators as GA: crossover, mutation and selection. The only difference is that GA relies on crossover where DE relies on mutation operation. DE algorithm uses mutation operation as a search mechanism and selection operation to direct the search in the search space as shown in Equation (14) and Equation (15). By creating trial vectors using the components of existing individuals in the population, the crossover operator effectively sorts information about successful combinations, enabling better solution search space [15].

$$\text{Mutation} \quad \hat{x}_i = x_{r1} + F(x_{r3} - x_{r2}), F = [0,1] \quad (14)$$

$$x_{r1}, x_{r2}, x_{r3} | r_1 \neq r_2 \neq r_3 \neq i \quad (15)$$

$$\text{Crossover} \quad y_i^j = \begin{cases} \hat{x}_i^j, R_j \leq CR \\ x_i^j, R_j > CR \end{cases}, R_j = [0,1] \quad (16)$$

During mutation, the parameter \hat{x}_i is mutant solution vector, while F is scaling factor and i is an index of current solution. In the stage of crossover, CR is the crossover constant, while j represent the j^{th} component of the corresponding array. In DE, a population of solution vectors is randomly created at the start. This population is successfully improved by applying mutation, crossover and selection operators as shown in Figure 3.4. In DE algorithm, each new solution produced competes with a mutant vector and the better one wins the competition. In other words, the chance of succession is independent on their fitness values. Every new solution produced competes with its parent and the better one wins the competition [15].

-
- 1: Initialization
 - 2: Evaluation
 - 3: **repeat**
 - 4: Mutation
 - 5: Recombination
 - 6: Evaluation
 - 7: Selection
 - 8: **until** convergence
-

Figure 3.4: General structure of DE

In this section, IA is being compared with the variants modified based on of DE, which are JDE and SADE. The IA outperforms JDE and SADE in most of the benchmark functions of Test 1 and Test 2. The statistical results in Table 3.6 and Table 3.7 indicate that IA performs better than JDE and SADE.

3.9 BSA vs IA

In BSA, three basic genetic operators – selection, mutation and crossover – are used to generate trial individuals. A random mutation strategy is performed such that only one direction individual is used for each target individual. BSA randomly chooses the direction

individual from a randomly chosen individual from previous generation. BSA uses a non-uniform crossover strategy that is more complex as compared with other GAs [39]. The procedure of BSA is as shown below.

-
- 1: Initialization
 - 2: **repeat**
 - 3: Selection I
 - 4: Mutation
 - 5: Crossover
 - 6: Selection II
 - 7: **until** convergence
-

Figure 3.5: General structure of BSA

BSA is divided into five processes: initialization, selection I, mutation, crossover and selection II. In the Selection II stage, the T_i s that have better fitness values than the corresponding P_i s are used to update the P_i s based on the concept of greedy selection. If the best individual of P (P_{best}) has better fitness value than the global minimum value, the global minimizer is updated to be P_{best} . Hence, the global minimum value is updated to be the fitness value of P_{best} .

$$\text{Mutation} \quad \text{Mutant} = P + F(\text{old}P - P) \quad (17)$$

$$\text{Crossover} \quad \text{map}_{n,m} = 1, T_{n,m} := P_{n,m} \begin{cases} n \in \{1,2,3,\dots,N\} \\ m \in \{1,2,3,\dots,D\} \end{cases} \quad (18)$$

The proposed IA is compared with BSA. The results indicate that IA works equally well as BSA in Test 1 and Test 2 as shown in Table 3.6 and 3.7. However, Table 3.8 shows that BSA is better than IA. The unique mutation and crossover strategies of BSA make it a powerful minimization technique. However, results of Test 1 and Test 2 denote that IA has higher convergence rate as compared with BSA because of the relatively simpler structure of IA.

4. Conclusions and Future Directions

In this work, a novel socio-inspired algorithm referred to as Party Ideology Algorithm (IA), which is mainly inspired from the human society individuals following certain ideology,

is proposed. Several operators were proposed and mathematically modelled for equipping the IA with high exploration and exploitation. The performance of the proposed algorithm was benchmarked on 75 test functions in terms of exploration, exploitation, local optima avoidance, fitness improvement of the population, and convergence rate. It can be concluded that the proposed algorithm benefits from high exploitation and convergence rate.

The IA is compared to seven well-known and recent algorithms: PSO, CMAES, ABC, JDE, CLPSO, SADE, and BSA. Wilcoxon statistical tests were also conducted when comparing the algorithms. The results showed that the proposed algorithm outperforms other algorithms in the majority of test functions. The statistical tests proved that the results were statistically significant for the IA. Thus, it may be concluded from the results that the proposed IA is comparable with other algorithms. Also, it is able to be applied as alternative optimizer for different optimization problems.

It is concluded that the IA improves the overall fitness of random initial solutions on optimization problems from the overall individuals' fitness. IA effectively searches and converges towards promising search space. Thus, the proposed algorithm is able to discover different regions of an optimization problem. Other remarks that can be made from the results of this study are as follows:

- Initial random walks of individuals around the parties emphasize exploration of the search space around the individuals.
- Effective in local optima avoidance since IA employs a population of search agents to approximate the global optimum.
- Promising search spaces are ensured since individuals relocate to the position of the best individuals during optimization.
- The best individual from each iteration is saved and considered as the elite, so all individuals tend towards the best solution obtained so far as well.
- IA has very few parameters to adjust. Thus it is a flexible algorithm for solving diverse class of problems.
- The unique mechanism of IA where the local party leader competes with every other party leader and the second best individual in its own party. This motivates the party leaders to explore a greater and promising search space. Also, they continuously look for a better solution in its own local neighbourhood.
- Every individual in every party to directly and indirectly compete with the same party individuals as well as other party individuals. This makes every party to remain in competition and grow which motivates the individuals search for better solutions.

Several research directions can be recommended for future studies with the proposed algorithm. Extending this algorithm to solve multi-objective problems can be considered. In addition, the algorithm could be applied for solving real world problems from healthcare as well as supply-chain disruption domain [52].

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