# Fuzzy C-Means with Wavelet Filtration for MR Image Segmentation

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***Abstract*—In this paper, we present an image segmentation technique based on fuzzy c-means (FCM) incorporated with wavelet domain noise filtration. With the use of image noise feature estimation composed of preliminary coefficient classification and wavelet domain indicator, a filter for balancing the preservation of relevant details against the degree of noise reduction can be created. The filter is further incorporated with FCM algorithm into the membership function for clustering. This approach allows FCM not only to exploit useful spatial information, but also dynamically minimize clustering errors caused by common noise in medical images. Experimental results suggest its usefulness for reducing FCM clustering noise sensitivity. In MR image segmentation applications, the proposed method outperforms other FCM variations, in terms of quantitative performance measure and visual quality.**

***Keywords*-fuzzy c-means; clustering; wavelet; noise reduction; segmentation; MR image**

## I. Introduction

Fuzzy c-means (FCM) clustering [1], [2], [3], [4] is a widely used unsupervised technique for feature analysis, clustering, and classifier designs in various areas. One of its successful applications is image segmentation where images can be represented in various feature spaces. FCM algorithm separates the image data by grouping similar data points in the feature space into clusters. This clustering is achieved by iteratively minimizing a cost function, which is dependent on the distance of the pixels to the cluster centroids in the feature domain [5], [6], [7]. FCM is especially useful for medical image segmentation due to its capacity of processing multiple-feature inputs, which can be created by the simultaneously generated multiple images from the multi-spectral imaging techniques [6], [8].

The image pixels in the immediate neighbors possess relatively similar feature data, therefore, the probability that adjacent pixels belong to the same cluster is high. Since the conventional FCM algorithm does not fully utilize this information, as a result, a noisy pixel can be wrongly classified because of its abnormal feature data. Therefore, researchers have proposed various methods for incorporating spatial information into FCM [9], [10], [11], [12], [13], where mean [9], [11], [10], Gaussian [12] and bilateral filters [13] are applied, respectively. With this approach, the membership weighting of each cluster is altered after the cluster distribution in the neighborhood is considered. This scheme reduces the effect of noise and biases the algorithm toward homogeneous clustering.

Although the incorporation of filters greatly enhances the fuzzy clustering performance, the filters applied in these works only on noise of limited frequency bandwidth, thus leaving noise of certain frequency untreated. To address this issue, Xiao et al. [14] extended the approach by taking advantage of a multi-resolution bilateral (MRB) filter to allow the improved FCM algorithm to reduce noise of both high and low-frequency. However, noise in the magnetic resonance (MR) image magnitude is Rician [15] which has a signal dependent mean, rather than Gaussian noise; and in ultrasound images, speckle noise may contain information useful to medical experts [16]. Due to the fact that MRB filter is not modeled to deal with noise for medical images, performance of this fuzzy clustering algorithm [14] on noise corrupted medical images is not high. As can be concluded, all above-mentioned techniques are less effective for medical image segmentation.

The aim of this study is to introduce a more flexible unsupervised image segmentation method for fuzzy clustering which not only include spatial information and preserve edges, but also adapt to various and unknown types of image noise. Our new method incorporates spatial information along with a wavelet domain indicator [17], and the membership weighting of each cluster is altered after the cluster distribution in the neighborhood is considered. Through quantitative evaluation and visual observation, we demonstrate its usefulness of reducing the effect of noise and biasing the algorithm toward homogeneous clustering on artificial and medical images.

## II. METHOD

### A. FCM Algorithm

The FCM algorithm generates prototypes and partitions for any set of numerical data by using fuzzy memberships [3]. In image segmentation application, let $X = (x_1; x_2; ...; x_N)$ denotes an image with $N$ pixels to be partitioned into $c$ clusters, where $x_i$ represents multidimensional data. The objective function of FCM algorithm can be described as follows:

$$\left\{ \begin{array}{l} J_{FCM} = \sum_{j=1}^{N} \sum_{i=1}^{c} \mu_{ij}^m d_{ij}^2 = \sum_{j=1}^{N} \sum_{i=1}^{c} \mu_{ij}^m \parallel x_j - a_i \parallel^2 , \\ \sum_{i=1}^{c} \mu_{ij} = 1, \forall j = 1, 2, 3, ..., N . \end{array} \right. \quad (1)$$

where $\mu_{ij} \in [0, 1]$ represents the membership degree of pixel $x_j$ in the $i$th cluster; $a_i$ is the centroid of cluster $i$; $m > 1$ is the fuzzy index which controls the fuzziness of the resulting partition; $d_{ij}^2 = ||x_j - a_i||^2$ is the Euclidean distance between pixel $x_j$ to centroid $a_i$.

The membership functions and cluster centroids update as shown respectively in Equation (2) and Equation (3):

$$\mu_{ij} = [\sum_{k=1}^{c} (\frac{||x_j - a_i||}{||x_j - a_k||})^{\frac{2}{m-1}}]^{-1}; 1 \leq j \leq N, 1 \leq i \leq c . \quad (2)$$

$$a_i = \frac{\sum_{j=1}^{N} \mu_{ij}^m x_j}{\sum_{j=1}^{N} \mu_{ij}^m}; 1 \leq i \leq c . \quad (3)$$

The general procedure of FCM algorithm is shown by following [4]:

1) Determine value of $c$, $m$ and converging error $\varepsilon$(in this paper, $\varepsilon = 0.00001$). Randomly choose an initial membership matrix $\mu^{(0)}$ with the constraint of Equation (1). Then at step $k, k = 1, 2, 3, ..., LMAX$.
2) Compute cluster centroids $a_i^{(k)}, i = 1, 2, 3, ..., c$ with Equation (3).
3) Compute an updated membership matrix $\mu^{(k+1)} = [\mu_{ij}^{(k+1)}]$ with Equation (2).
4) Compare $\mu^{(k)}$ to $\mu^{(k+1)}$ or $a^{(k)}$ to $a^{(k+1)}$ or $J^{(k)}$ to $J^{(k+1)}$ in any convenient norm. If the changes are less than $\varepsilon$, stop. Otherwise, set $\mu^{(k)} = \mu^{(k+1)}$ and return to step 2.

### B. Wavelet Denoising Filter

Noise corrupted image can be modeled as $Y = A + N$, where $A$ is the noiseless image, $N$ is the noise and $Y$ is the test image [18]. Mean of $Y$ can be retrieved according to Equation (4) as follows:

$$E(Y) = \sigma \sqrt{\frac{\pi}{2}} M(-\frac{1}{2}, 1, -\frac{A^2}{2\sigma^2}) \quad (4)$$

where $(M(, , , )$ is the confluent hypergeometric function [19]. Since it was pointed out that the square of Y is the sum of the square of A and the square of noise [15], $\widehat{Y}$ can be calculated by $\widehat{Y^2} = Y^2 - 2\widehat{\sigma}^2$.

Then the multi-resolution method is applied to decompose the image into 2 levels and a non-linear shrinkage process on wavelet coefficients of each level and orientation is performed. This process contains several steps of establishing the significance labels, producing the histogram based on these labels, estimating the probability density functions, and shrinking the wavelet coefficients. The significant label can hence be calculated with the assistance of the generalized likelihood ratio method (see Equation (5)).

$$\hat{x}_s = \frac{1}{1 + \eta(y_s)\xi} y_s \quad (5)$$

where $\eta(y_s) = \frac{p_{Y|H}(y_s|H_0)}{p_{Y|H}(y_s|H_1)}$ and $\xi = \frac{P(H_0)}{P(H_1)}$. The probability density function can then be estimated by the histogram of $S_0 = \{l : \widehat{x}_l = 0\}$ and $S_1 = \{l : \widehat{x}_l = 1\}$. In order to reduce the effect of the errors in the tails, the log function [17] is applied to fit the distribution. The result of the value then becomes $\widehat{y}_k = \frac{\widehat{r}\widehat{\xi_k}\widehat{\eta}_k}{1+\widehat{r}\widehat{\xi_k}\widehat{\eta}_k}\omega_k$, where $\widehat{r}$ is the ratio of the number of significant labels to the non-significant labels [17]. Finally, the inverse wavelet filter can be applied to restore the image.

### C. Incorporating FCM with Wavelet Denoising Filter

In the FCM algorithm, the membership function based on the wavelet noising filter is firstly calculated. By defining the indicator function as: $Indicator_i = \frac{\sum_{k \in N}(x_i)(label_{x_k} == label_{x_i})}{Size of N}$, where $N(x_i)$ represents a square window centered on pixel $x_i$ in the spatial domain, $label_x$ represents the index of the maximum membership value. With this, the image pixel will be replaced by its value convolved with a Gaussian filter, in the cases where the indicator function is less than $p$, thus eliminating the noise in the image. In the rest of the paper, the proposed algorithm is referred to as wFCM.

The outline of the proposed algorithm can be illustrated as in Fig. 1.

## III. EXPERIMENTS

### A. Image Data

In this study, synthetic images and multi-spectral brain MR images [20] are used. All the image data used in the experiment is created by combining two images (dark and bright images for synthetic images; T1- and T2-weighted M-R images for medical images), thus creating a 2-dimensional matrix as the input data for fuzzy clustering algorithm. To evaluate noise sensitivity of fuzzy clustering algorithms, the

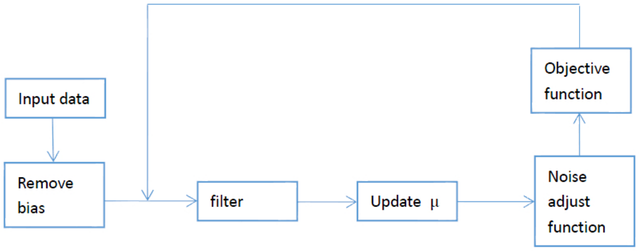


Fig. 1. Outline of the FCM with wavelet denoising algorithm.

images are corrupted by artificial Rician noise with various noise ratio and respective peak signal-to-noise ratio(PSNR) value.

### B. Clustering Validity Functions

Two types of clustering validity functions, fuzzy partition and feature structure, are often used to evaluate the performance of clustering. The validity functions based on fuzzy partition are partition coefficient $V_{pc}$ [21] and partition entropy $V_{pe}$ [22]. They are respectively defined as follows:

$$V_{pc} = \frac{\sum_{j=1}^{N}\sum_{i=1}^{c}\mu_{ij}^{2}}{N} \tag{6}$$

$$V_{pe} = \frac{-\sum_{j=1}^{N}\sum_{i=1}^{c}\mu_{ij}\log\mu_{ij}}{N} \tag{7}$$

The best clustering is achieved when the value $V_{pc}$ is maximal or $V_{pe}$ is minimal, it expresses that less fuzziness means better performance.

Another kind of validity function is based on the feature structure [23], [24], it does not only take the fuzzy partition into account, but also make a direct connection to the featuring property. $V_{xb}$ is widely used one of this kind of validity functions and defined as Equation (8).

$$V_{xb} = \frac{\sum_{j=1}^{N}\sum_{i=1}^{c}\mu_{ij}^{m}||x_j - a_i||^2}{N \times (min_{1\leq k,m\leq c,m\neq k}||a_k - a_m||^2)} \tag{8}$$

A good clustering result generates samples that are compact within one cluster and samples that are separated between different clusters. Minimizing $V_{xb}$ is expected to lead to a good clustering [23].

## IV. Results and Discussion

### A. Synthetic Image Segmentation

Fig. 2 shows the original synthetic and the correspondingly noise-corrupted images. Fig. 3 visualizes the clustering results by the used methods. In the first row where no noise is added, all the clustering techniques create desirable segmentation results that edges of segments are generally smooth, whereas in the second and third rows, the added noise significantly corrupts the clustering results in the case of FCM; for the clustering results by the use of sFCM,

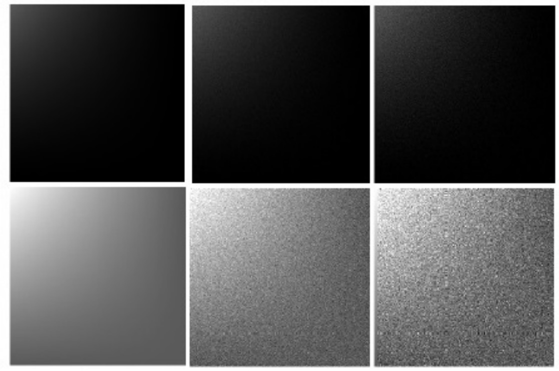

Fig. 2. Synthetic and their noise-corrupted images: a pair of original images (first column), images respectively corrupted by noise ratio of 5% (second column), and by noise ratio of 10% (third column)

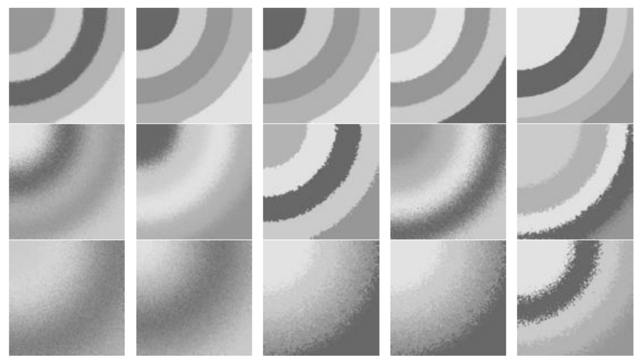

Fig. 3. Visualized clustering results on images from Fig. 2: original images(first row), images corrupted by noise ratio of 5% (second row) and images corrupted by noise ratio of 10% (third row), respectively by: FCM (first column), sFCM (second column), gFCM (third column), bFCM (fourth column) and wFCM (fifth column)

gFCM and bFCM as visualized respectively in the second, third and forth columns, the rough shape of clusters are shown though edges are still not smooth. It can be seen that, in the fifth column, the clustering results outlines fairly smooth as shown in the second row when noise ratio of 5% (shown in the second row) is applied. Even in the case when the strong noise ratio of 10% (shown in the third row) does not significantly alter the clustering edge shape. The blurred edges created by sFCM, gFCM and bFCM are replaced by smooth and clear curves in the case of wFCM clustering. This clustering performance improvement can be attributed to the combination of FCM membership function and wavelet filter which biases clustering solution toward homogeneous grouping.

### B. Brain MR Image Segmentation

Fig. 4(c-h) shows the clustering results on brain MR image corrupted by strong Rician noise ratio of 20%. The results show that all the tested clustering techniques creates desirable segmentation results. It can be observed that, the

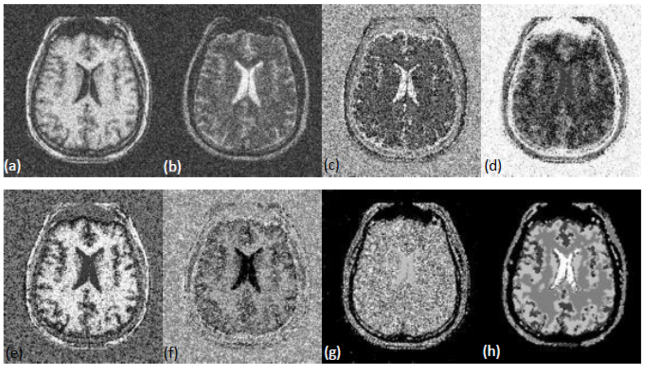


Fig. 4. Input images: (a) T1- and (b) T2-weighted and visualized clustering results by: (c) FCM; (d) sFCM; (e) gFCM; (f) bFCM; (g) Wavelet Filter + FCM; (h) wFCM

Table I
AVERAGE (EXECUTION OF EACH ALGORITHM FOR 5 TIMES) VALIDITY FUNCTION RESULTS OBTAINED BY VARIOUS FCM TECHNIQUES USING IMAGE DATA SHOWN IN FIG. 4(A) AND (B).

| | FCM | sFCM | gFCM | bFCM | wFCM |
|---|---|---|---|---|---|
| $V_{pc}$ | 0.74 | 0.77 | 0.80 | 0.868 | 0.88 |
| $V_{pe}$ | 0.51 | 0.44 | 0.39 | 0.26 | 0.22 |
| $V_{xb}$ | 0.38 | 0.36 | 0.30 | 0.279 | 0.22 |

spurious blobs in the segmentation result by wFCM are fully removed, while all other FCM variants create less homogeneous clustering results. In this case, wFCM outperforms all other techniques, thus making the segmentation more related to the actual anatomical meaning of the images. The clustering result with wavelet denoising filter followed by conventional FCM is also tested in Fig. 4(g), although this approach remove spurious noise points in the background region, comparative visual observation with Fig. 4(h), where wFCM is applied, further removes noisy points in the brain region and creates more homogeneous clustering result.

Table I tabulates the validity functions used to evaluate the performance of FCM clustering for MR images. The validity functions of both $V_{pc}$ and $V_{pe}$ of wFCM is the best among all the tested techniques in this work. This justifies effect on the membership function $\mu$ in the clustering from the included wavelet filter. Clustering fuzziness caused by Rician noise is reduced by the filter, whereas filters included in other FCM variants work only on removing general image signal.

## V. CONCLUSION

The proposed wFCM clustering method for medical image segmentation is designed to reduce the Rician noise which is common in medical imaging. By incorporating specific wavelet filter, the conventional FCM is capable of handling Rician distributed signal more efficiently and effectively. The noise adjust functions are also utilized to detect the missing noise and correct it with Gaussian filter. This step could compensate the disadvantage of the wavelet filters that several patterns of noise could be treated as signal. With the help of this step, performance of the proposed method can be higher than the conventional FCM and other kernel-based FCM variants.

This method was tested on both synthetic and MR images. The conventional FCM and other existing FCM variants of similar approach are tested. Since the Rician distributed signal is different from the Gaussian noise when the SNR is low, the proposed method could take advantage of this and outperform other methods especially when the noise ratio is high. In addition to the subjective observation, validity functions are applied for conducting discriminative and analytical experiments to evaluate the performance and robustness of the proposed method. The experimental results show the proposed method has superior performance over all existing filters included modified FCM algorithms. This technique also creates better performance than using the wavelet denoising filter followed by conventional FCM clustering.

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